

Algebra

CHAPTER 9

Probability I

9.1–9.20

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9

Probability I

SOME DEFINITIONS

EXPERIMENT

An operation which results in some well-defined outcomes is called an experiment.

RANDOM EXPERIMENT

An experiment whose outcome cannot be predicted with certainty is called a random experiment. In other words, if an experiment is performed many times under similar conditions and the outcome each time is not the same, then this experiment is called a random experiment.

Examples of random experiment are:

- “Tossing a fair coin” is a random experiment because on tossing a coin, either a head or a tail will come up. Also, if we toss a coin again and again the outcome each time will not be the same.
- “Throwing an unbiased dice” is a random experiment because when a dice is thrown, we cannot say with certainty which one of the numbers 1, 2, 3, 4, 5 and 6 will come up.

An experiment whose outcome can be foretold before is not a random experiment. For example, when a stone is thrown upwards it is sure that the stone will fall downward. Therefore, throwing a stone upward is not a random experiment.

SAMPLE SPACE

The set of all possible outcomes of a random experiment is called the sample space for that experiment. It is usually denoted by S .

Examples of sample space are:

- When a coin is tossed either a head or a tail will come up. If H denotes the occurrence of head and T denotes the occurrence of tail, then sample space, $S = \{H, T\}$.
- When two coins are tossed simultaneously, sample space is given by $S = \{(H, H), (H, T), (T, H), (T, T)\}$, where (H, H) denotes the occurrence of head on the first coin and occurrence of head on the second coin. Similarly, (H, T) denotes the occurrence of head on the first coin and occurrence of tail on the second coin.
- When a dice is thrown, any one of the numbers 1, 2, 3, 4, 5 and 6 will come up. Therefore, sample space, $S = \{1, 2, 3, 4, 5, 6\}$.

Here, 1 denotes the occurrence of 1, 2 denotes the occurrence of 2 and so on.

- When two balls are drawn from a bag containing 3 red balls R_1, R_2, R_3 and 2 black balls B_1, B_2 , sample space is given by

$$S = \{(R_1, R_2), (R_1, R_3), (R_2, R_3), (B_1, B_2), (R_1, B_1), (R_1, B_2), (R_2, B_1), (R_2, B_2), (R_3, B_1), (R_3, B_2)\}$$

EVENT

Consider the experiment of tossing a coin two times. An associated sample space is $S = \{HH, HT, TH, TT\}$.

Now, suppose that we are interested in those outcomes which correspond to the occurrence of exactly one head. We find that HT and TH are the only elements of S corresponding to the occurrence of this happening (event). These two elements form the set $E = \{HT, TH\}$. Here, E is the event that exactly one head shows.

Here, set E is a subset of the sample space S . Similarly, we find the following correspondence between events and subsets of S .

Description of events	Corresponding subset of 'S'
Number of tails is exactly two	$\{TT\}$
Number of tails is at least one	$\{HT, TH, TT\}$
Number of heads is at most one	$\{HT, TH, TT\}$
Second toss is not head	$\{HT, TT\}$
Number of tails is at most two	$\{HH, HT, TH, TT\}$
Number of tails is more than two	ϕ

When a dice is thrown, sample space $S = \{1, 2, 3, 4, 5, 6\}$

Let $A = \{1, 3, 5\}$, $B = \{5, 6\}$ and $C = \{2, 3, 5\}$

Here, A is the event of occurrence of an odd number, B is the event of the occurrence of a number greater than 4 and C is the event of occurrence of a prime number.

Note:

- Sample space S plays the same role as the universal set for all problems related to the particular experiment.
- ϕ is also a subset of S which is called an impossible event.
- S is also a subset of S which is called a sure event or a certain event.

The above discussion suggests that a subset of sample space is associated with an event and an event is associated with a subset of sample space. In the light of this, we define different cases of events as follows:

Simple Event or Elementary Event

If an event E has only one sample point of a sample space, it is called a simple (or elementary) event.

Few examples of simple event are:

- (i) When a coin is tossed, sample space $S = \{H, T\}$.
 Let $A = \{H\}$ = the event of occurrence of head
 and $B = \{T\}$ = the event of occurrence of tail
 Here, A and B are simple events.
- (ii) When a coin is tossed two times, sample space
 $S = \{HH, HT, TH, TT\}$.
 Let $A = \{TT\}$ = the event of occurrence of two tails
 $B = \{TH\}$ = the event of occurrence in which results
 of first and second tosses are T and H , respectively.
 Here, A and B are simple events.
- (iii) When a dice is thrown, sample space $S = \{1, 2, 3, 4, 5, 6\}$.
 Let $A = \{5\}$ = the event of occurrence of 5.
 $B = \{2\}$ = the event of occurrence of 2.
 Here, A and B are simple events.

Mixed Event or Compound Event or Composite Event

A subset of the sample space S which contains more than one element is called a mixed event.

For example, in the experiment of “tossing a coin thrice” the events

- E : ‘Exactly one head appeared’
 F : ‘At least one head appeared’
 G : ‘At most one head appeared’ etc.

are all compound events.

The subsets of S associated with these events are:

- $E = \{HTT, THT, TTH\}$
 $F = \{HTT, THT, TTH, HHT, HTH, THH, HHH\}$
 $G = \{TTT, THT, HTT, TTH\}$

Each of the above subsets contains more than one sample points, hence they are all compound events.

Trial

When an experiment is repeated under similar conditions and it does not give the same result each time but may result in any one of the several possible outcomes, the experiment is called a trial and the outcomes are called cases. The number of times the experiment is repeated is called the number of trials. For example, one toss of a coin is a trial when the coin is tossed five times and one throw of a dice is a trial when the dice is thrown four times.

DIFFERENT TYPES OF EVENTS

Complementary Event

For every event A , there corresponds another event A' called the complementary event to A . It is also called the event ‘not A ’.

For example, take the experiment ‘of tossing three coins’. An associated sample space is

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Let $A = \{HTH, HHT, THH\}$ be the event ‘only one tail appears’

Clearly, for the outcome HTT , the event A has not occurred. But we may say that the event ‘not A ’ has occurred. Thus, for every outcome which is not in set A , we say that ‘not A ’ occurs.

Thus, the complementary event ‘not A ’ to the event A is

$$A' = \{HHH, HTT, THT, TTH, TTT\}$$

Thus, $A' = \{\omega : \omega \in S \text{ and } \omega \notin A\} = S - A$.

A' is also denoted by A^c or $\bar{A}$.

The Event ‘A or B’

Recall that union of two sets A and B denoted by $A \cup B$ contains all those elements which are either in A or in B or in both. When the sets A and B are two events associated with a sample space, then ‘ $A \cup B$ ’ is the event ‘either A or B or both’ also called sometimes ‘ A or B ’.

The Event ‘A and B’

We know that intersection of two sets $A \cap B$ is the set of those elements which are common to both A and B . i.e., which belong to both ‘ A and B ’. If A and B are two events, then the set $A \cap B$ denotes the event ‘ A and B ’.

The Event ‘A but not B’

We know that $A - B$ is the set of all those elements which are in A but not in B . Therefore, the set $A - B$ denotes the event ‘ A but not B ’. Also, $A - B$ is same as $A \cap B'$ or $A - (A \cap B)$.

Equally Likely Events

Cases (outcomes) are said to be equally likely when we have no reason to believe that one is more likely to occur than the other. Thus, when an unbiased dice is thrown all the six faces 1, 2, 3, 4, 5 and 6 are equally likely to come up. Similarly, when an unbiased coin is tossed occurrence of head and tail are equally likely cases.

Exhaustive Cases (Events)

Consider the experiment of throwing a dice. We have $S = \{1, 2, 3, 4, 5, 6\}$. Let us define the following events:

- A : ‘a number less than 4 appears’,
 B : ‘a number greater than 2 but less than 5 appears’

And C : ‘a number greater than 4 appears’

Then $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{5, 6\}$. We observe that

$$A \cup B \cup C = \{1, 2, 3\} \cup \{3, 4\} \cup \{5, 6\} = S$$

Such events A , B and C are called exhaustive events.

In general, if $E_1, E_2, \dots, E_n$ are n events of a sample space S and if $E_1 \cup E_2 \cup \dots \cup E_n = \bigcup_{i=1}^n E_i = S$, then $E_1, E_2, \dots, E_n$ are called

exhaustive events. In other words, events $E_1, E_2, \dots, E_n$ are said to be exhaustive if at least one of them necessarily occurs whenever the experiment is performed.

Mutually Exclusive Events

In the experiment of rolling a dice, a sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

Consider events, A ‘an odd number appears’ and B ‘an even number appears’.

Clearly, the event A excludes the event B and vice versa.

In other words, there is no outcome which ensures the occurrence of events A and B simultaneously.

Here, $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$.

Clearly, $A \cap B = \phi$, i.e., A and B are disjoint sets.

In general, two events A and B are called mutually exclusive events if the occurrence of any one of them excludes the occurrence of the other, i.e., if they can not occur simultaneously.

Again, in the experiment of rolling a dice, consider the events A 'an odd number appears' and event B 'a number less than 4 appears'

Obviously, $A = \{1, 3, 5\}$ and $B = \{1, 2, 3\}$. Here, $A \cap B = \{1, 3\} \neq \phi$.

Therefore, A and B are not mutually exclusive events.

ILLUSTRATION 9.1

A coin is tossed three times, consider the following events.

A : 'No head appears',

B : 'Exactly one head appears' and

C : 'At least two heads appear'.

Do they form a set of mutually exclusive and exhaustive events?

Sol. The sample space of the experiment is

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

and $A = \{TTT\}$,

$$B = \{HTT, THT, TTH\},$$

$$C = \{HHT, HTH, THH, HHH\}$$

$$\text{Now, } A \cup B \cup C = \{TTT, HTT, THT, TTH, HHT, HTH, THH, HHH\} \\ = S$$

Therefore, A , B and C are exhaustive events.

Also, $A \cap B = \phi$, $A \cap C = \phi$ and $B \cap C = \phi$

Therefore, the events are pair-wise disjoint, i.e., they are mutually exclusive.

Hence, A , B and C form a set of mutually exclusive and exhaustive events.

AXIOMATIC APPROACH TO PROBABILITY

Probability is the measure of the likelihood that an event will occur. Probability is quantified as a number between 0 and 1, where, 0 indicates impossibility and 1 indicates certainty. The higher the probability of an event, the more likely it is that the event will occur. If a fair coin is tossed, two outcomes ("Head" and "Tail") are both equally probable. We can say there are 50% chances that head will appear and 50% chances that tail will appear. So, the probability of appearing a head is equal to the probability of appearing a tail; and since no other outcomes are possible, the probability of each of "head" and "tail" is $1/2$.

There are different methods to find the value of the probability. Till now, we have been using statistical approach to find the probability which is based on observations and collected data.

Axiomatic approach is another way of describing probability of an event.

In this approach, some axioms or rules are depicted to assign probabilities.

Let S be the sample space of a random experiment.

The probability P is a real valued function whose domain is the power set of S and range is the interval $[0,1]$ satisfying the following axioms:

- For any event E , $P(E) \geq 0$, where $P(E)$ is probability value or simply probability of event E .

- $P(S) = 1$

- If E and F are mutually exclusive events, then

$$P(E \cup F) = P(E) + P(F).$$

- It follows from (iii) that $P(\phi) = 0$

Let S be a sample space containing outcomes $\omega_1, \omega_2, \dots, \omega_n$.

$$\therefore S = \{\omega_1, \omega_2, \dots, \omega_n\}$$

It follows from the axiomatic definition of probability that

- $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$

- $P(\omega_1) + P(\omega_2) + \dots + P(\omega_n) = 1$

- For any event A , $P(A) = \sum P(\omega_i)$, $\omega_i \in A$.

Consider the experiment in which coin is tossed only once. Then $S = \{H, T\}$. Since $P(S) = 1$, and the events $\{H\}$ and $\{T\}$ are mutually exclusive, we have $P(\{H\}) + P(\{T\}) = P(\{H\} \cup \{T\}) = P(S) = 1$.

If the coin is fair, we should assign 0.5 to $P(\{H\})$ and 0.5 to $P(\{T\})$.

If the coin is more likely to give a head, then 0.8 for $P(\{H\})$ and 0.2 for $P(\{T\})$ may be suitable.

In fact, if p is any fixed number between 0 and 1, then $P(\{H\}) = p$, and $P(\{T\}) = 1 - p$.

Probabilities of Equally Likely Outcomes

Let a sample space of an experiment be $S = \{\omega_1, \omega_2, \dots, \omega_n\}$.

Let all the outcomes are equally likely to occur, i.e., the chances of occurrence of each simple event must be the same.

So, $P(\omega_i) = p$, for all $\omega_i \in S$ where $0 \leq p \leq 1$

Since $\sum_{i=1}^n P(\omega_i) = 1$,

$$p + p + p + \dots + p \text{ (n times)} = 1$$

$$\therefore np = 1$$

Therefore, $p = \frac{1}{n}$

Let S be a sample space and E be an event, such that $n(S) = n$ and $n(E) = m$. If each outcome is equally likely, then it follows that

$$P(E) = \frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n} \text{ (m times)} = \frac{m}{n}$$

ILLUSTRATION 9.2

Find the probability of getting the sum more than 7 when two dices are rolled simultaneously.

Sol.

Sum of numbers on two dices	Numbers on two dices	Number of cases
8	(4, 4), (3, 5), (5, 3), (2, 6), (6, 2)	5
9	(4, 5), (5, 4), (3, 6), (6, 3)	4
10	(5, 5), (4, 6), (6, 4)	3
11	(5, 6), (6, 5)	2
12	(6, 6)	1

If $P(i)$ is probability of event in which sum of the numbers on two dices is ' i ' then

Required probability is,

$$P(8) + P(9) + P(10) + P(11) + P(12) \\ = \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{15}{36} = \frac{5}{12}$$

ILLUSTRATION 9.3

A dice is loaded so that the probability of a face i is proportional to i^2 , $i = 1, 2, \dots, 6$. Then find the probability of occurring a prime number when the dice is rolled.

Sol. Let event A_i is “number i appears”

$\therefore P(A_i) \propto i^2$ or $P(A_i) = \lambda i^2$, where λ is constant of proportionality.

$$\text{Now, } \sum_{i=1}^6 P(i) = 1$$

$$\therefore \sum_{i=1}^6 \lambda i^2 = 1$$

$$\Rightarrow \lambda \times \frac{6 \times 7 \times 13}{6} = 1$$

$$\Rightarrow \lambda = \frac{1}{91}$$

$$\therefore P(A_i) = \frac{i^2}{91}$$

$$\therefore P(\text{prime number appears}) = P(2) + P(3) + P(5) \\ = \frac{2^2}{91} + \frac{3^2}{91} + \frac{5^2}{91} = \frac{38}{91}$$

ILLUSTRATION 9.4

Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a dice. Find the probability of

- (i) the dice shows a number greater than 4
- (ii) there is at least one tail

Sol. According to the question, sample space

$$S = \{(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$$

In the above set, there are 8 elementary events, but all are not equally likely.

However, events (H, H) and (H, T) are equally likely.

Each event has probability $\frac{1}{4}$ (considering sample space $\{HH, HT, TH, TT\}$)

Events $(T, 1), (T, 2), \dots, (T, 6)$ are also equally likely.

Let probability of each be p . Then

$$6p = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$$

$$\therefore p = \frac{1}{12}$$

Let event A be “the dice shows a number greater than 4”, and event B be “there is at least one tail”.

$$\therefore A = \{(T, 5), (T, 6)\}$$

$$\text{and } B = \{(H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$$

$$\therefore P(A) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$$

$$P(B) = \frac{1}{4} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

ILLUSTRATION 9.5

Four candidates A, B, C and D have applied for the post in government office. If A is twice as likely to be selected as B , and B and C are given about the same chances of being selected, while C is twice as likely to be selected as D , what are the probabilities that

- (i) C will be selected?
- (ii) A will not be selected?

Sol. It is given that A is twice as likely to be selected as B .

$$\therefore P(A) = 2P(B)$$

B and C have same chances of being selected.

$$\therefore P(B) = P(C)$$

C is twice as likely to be selected as D .

$$\therefore P(C) = 2P(D)$$

$$\text{So, } P(B) = 2P(D)$$

$$\Rightarrow P(A) = 4P(D) \Rightarrow P(D) = \frac{P(A)}{4}$$

Sum of all probabilities = 1

$$\therefore P(A) + P(B) + P(C) + P(D) = 1$$

$$\Rightarrow P(A) + \frac{P(A)}{2} + \frac{P(A)}{2} + \frac{P(A)}{4} = 1$$

$$\Rightarrow \frac{4P(A) + 2P(A) + 2P(A) + P(A)}{4} = 1$$

$$\Rightarrow 9P(A) = 4 \Rightarrow P(A) = \frac{4}{9}$$

$$(i) P(C \text{ will be selected}) = P(C) = \frac{P(A)}{2} = \frac{4}{9 \times 2} = \frac{2}{9}$$

$$(ii) P(A \text{ will not be selected}) = P(B \text{ or } C \text{ or } D \text{ will be selected}) \\ = P(B) + P(C) + P(D) \\ = \frac{P(A)}{2} + \frac{P(A)}{2} + \frac{P(A)}{4} = \frac{5}{9}$$

ILLUSTRATION 9.6

If $\frac{1+3p}{3}$, $\frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then find the set of all values of p .

$$\text{Sol. Let } P(A) = \frac{1+3p}{3}, P(B) = \frac{1-p}{4}, P(C) = \frac{1-2p}{2}$$

As A, B and C are three mutually exclusive events.

$$\therefore P(A) + P(B) + P(C) \leq 1$$

$$\Rightarrow \frac{1+3p}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \leq 1$$

$$\Rightarrow 4 + 12p + 3 - 3p + 6 - 12p \leq 12$$

$$\Rightarrow p \geq 1/3$$

$$\text{Also, } 0 \leq P(A) \leq 1$$

$$\Rightarrow 0 \leq \frac{1+3p}{3} \leq 1$$

(1)

$$\begin{aligned} 0 &\leq 1 + 3p \leq 3 \\ \Rightarrow -\frac{1}{3} &\leq p \leq \frac{2}{3} \end{aligned} \quad (2)$$

$$\begin{aligned} 0 &\leq P(B) \leq 1 \\ \Rightarrow 0 &\leq \frac{1-p}{4} \leq 1 \\ \Rightarrow 0 &\leq 1-p \leq 4 \\ \Rightarrow -3 &\leq p \leq 1 \end{aligned} \quad (3)$$

$$\begin{aligned} 0 &\leq P(C) \leq 1 \\ \Rightarrow 0 &\leq \frac{1-2p}{2} \leq 1 \\ \Rightarrow -\frac{1}{2} &\leq p \leq \frac{1}{2} \end{aligned} \quad (4)$$

Combining (1), (2), (3), and (4), we get

$$\frac{1}{3} \leq p \leq \frac{1}{2}$$

CONCEPT APPLICATION EXERCISE 9.1

1. Which of the following cannot be valid assignment of probabilities for outcomes of sample space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$?

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
(e)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

2. Consider the following assignments of probabilities for outcomes of sample space $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Number (X)	1	2	3	4	5	6	7	8
Probability, P(X)	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

Find the probability that

- (a) X is a prime number
 (b) X is a number greater than 4.
 3. Find the probability that a leap year will have 53 Fridays or 53 Saturdays.
 4. A dice is loaded so that the probability of a face i is proportional to i , $i = 1, 2, \dots, 6$. Then find the probability of an even number occurring when the dice is rolled.
 5. Find the probability of drawing either an ace or a king from a pack of card in a single draw.
 6. Three faces of a fair dice are yellow, two are red and one is blue. Find the probability that the dice shows (a) yellow, (b) red and (c) blue face.

ANSWERS

1. (c), (d) and (e) 2. (a) 0.62 (b) 0.4 3. $\frac{3}{7}$
 4. $\frac{4}{7}$ 5. $\frac{2}{13}$ 6. (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$

MATHEMATICAL OR CLASSICAL DEFINITION OF PROBABILITY

In axiomatic approach of the probability, we learned that for sample space S and event E , if $n(S) = n$, $n(E) = m$ and each outcome is equally likely, then

$$\begin{aligned} P(E) &= \frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n} \text{ (} m \text{ times)} \\ &= \frac{m}{n} \\ &= \frac{n(E)}{n(S)} = \frac{\text{number of elements in } E}{\text{number of elements in } S} \\ &= \frac{\text{number of cases favourable to event } E}{\text{total number of cases}} \end{aligned}$$

Few examples based on this are:

1. When a dice is rolled, sample space $S = \{1, 2, 3, 4, 5, 6\}$. Let A be the event of occurrence of an odd number. Then $A = \{1, 3, 5\}$. Also, let B be the event of occurrence of a number greater than 4. Then $B = \{5, 6\}$.

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2} \text{ and } P(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

2. When two coins are tossed, sample space $S = \{HH, HT, TH, TT\}$. Let E be the event of occurrence of one head and one tail. Then $E = \{HT, TH\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

3. When one ball is drawn at random from a bag containing 3 black and 4 red balls (balls of the same colour being identical or different), then sample space $S = \{B_1, B_2, B_3, R_1, R_2, R_3, R_4\}$.

$$\therefore n(S) = 7$$

Here, the three black balls may be denoted by B_1, B_2 and B_3 even if they are identical.

Let E be the event of occurrence of a red ball.

Then $E = \{R_1, R_2, R_3, R_4\}$. So, $n(E) = 4$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{7}$$

VALUE OF PROBABILITY OF OCCURRENCE OF AN EVENT

Let S be the sample space and E be an event.

Then $\phi \subseteq E \subseteq S$

$$\therefore n(\phi) \leq n(E) \leq n(S)$$

$$\Rightarrow 0 \leq n(E) \leq n(S)$$

$$\Rightarrow \frac{0}{n(S)} \leq \frac{n(E)}{n(S)} \leq \frac{n(S)}{n(S)}$$

[Dividing by $n(S)$]

$$\Rightarrow 0 \leq P(E) \leq 1 \quad \left(\because P(E) = \frac{n(E)}{n(S)} \right)$$

Thus, if ϕ is the impossible event, then $P(\phi) = \frac{n(\phi)}{n(S)} = \frac{0}{n(S)} = 0$

and if S be the sure event, then $P(S) = \frac{n(S)}{n(S)} = 1$.

VALUE OF PROBABILITY OF COMPLEMENT EVENT

If E is any event and E' be the complement of event E .

Since E and E' are disjoint and exhaustive sets,

$$E \cup E' = S$$

$$\therefore n(E \cup E') = n(S)$$

$$\Rightarrow n(E) + n(E') = n(S)$$

$$\Rightarrow \frac{n(E)}{n(S)} + \frac{n(E')}{n(S)} = 1$$

$$\Rightarrow P(E) + P(E') = 1$$

ODDS IN FAVOUR AND ODDS AGAINST AN EVENT

Let S be the sample space and E be an event.

Also, let E' denote the complement of event E .

Then

(i) Odds in favour of event

$$\begin{aligned} E &= \frac{n(E)}{n(E')} \\ &= \frac{\text{number of cases favourable to event } E}{\text{number of cases against } E} \end{aligned}$$

Also, odds in favour of event

$$E = \frac{n(E)}{n(E')} = \frac{n(E)/n(S)}{n(E')/n(S)} = \frac{P(E)}{P(E')}$$

(ii) Odds against event

$$\begin{aligned} E &= \frac{n(E')}{n(E)} \\ &= \frac{\text{number of cases against event } E}{\text{number of cases favourable to event } E} \end{aligned}$$

$$\text{Also, odds against event } E = \frac{n(E')}{n(E)} = \frac{P(E')}{P(E)}$$

If any one of $P(E)$, odds in favour of E and odds against E is given, then other two can be determined.

For example,

(i) If $P(E) = \frac{2}{7}$, then odds in favour of $E = \frac{2}{5}$ and odds against

$$E = \frac{5}{2}$$

(ii) If odds against $E = \frac{3}{11}$, then odds in favour of $E = \frac{11}{3}$ and

$$P(E) = \frac{11}{14}$$

(iii) If odds in favour of $E = \frac{3}{8}$, then odds against $E = \frac{8}{3}$ and

$$P(E) = \frac{3}{11}$$

ILLUSTRATION 9.7

A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. Find the probability that the determinant chosen is non-zero.

Sol. Determinant Δ of order 2 will be of the form $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

$$\therefore \Delta = ad - bc$$

The total number of ways of choosing a, b, c and d is $2 \times 2 \times 2 \times 2 = 16$.

Now, $\Delta \neq 0$ if and only if either $ad = 1, bc = 0$ or $ad = 0, bc = 1$.

For $ad = 1, (a, d) \equiv (1, 1)$.

For $bc = 0, (b, c) \equiv (0, 1), (1, 0), (0, 0)$.

So, three cases are there.

Similarly, for $ad = 0$ and $bc = 1$, there will be three cases.

$\therefore$ Number of favourable cases = $3 + 3 = 6$

$$\text{So, required probability} = \frac{6}{16} = \frac{3}{8}$$

ILLUSTRATION 9.8

A dice is rolled three times, find the probability of getting a larger number than the previous number each time.

Sol. Total number of cases = $6^3 = 216$.

If a larger number appears than the previous number each time, all the three numbers are distinct.

Three numbers can be selected from six numbers in ${}^6C_3 = 20$ ways and there is only one way in which three selected numbers can appear.

$$\text{Hence, required probability} = \frac{20}{216} = \frac{5}{54}$$

ILLUSTRATION 9.9

If a coin is tossed n times, then find the probability that the head appears odd number of times.

Sol. Total number of cases = 2^n

If head appears odd number of times (1, 3, 5, ... times), then

Number of favourable ways = $({}^nC_1 + {}^nC_3 + {}^nC_5 + \dots) = 2^{n-1}$

$$\therefore \text{Required probability} = \frac{2^{n-1}}{2^n} = \frac{1}{2}$$

ILLUSTRATION 9.10

A card is drawn at random from a pack of cards. What is the probability that the drawn card is neither a heart nor a king?

Sol. There are 13 cards which are heart and 4 cards which are king.

But one card is king of heart.

So, number of cards which are king or heart = $13 + 4 - 1 = 16$

Number of cards which are not king or heart = $52 - 16 = 36$

$$\text{So, required probability} = \frac{36}{52} = \frac{9}{13}$$

ILLUSTRATION 9.11

A card is drawn from a pack of 52 cards. A person bets that it is a spade or an ace. What are the odds against him of winning this bet?

Sol. Number of cards which are spade or ace = $13 + 4 - 1 = 16$
 So, probability of the card being a spade or an ace = $\frac{16}{52} = \frac{4}{13}$
 Hence, odds in favour is 4 : 9.
 Therefore, odds against him of winning the bet is 9 : 4.

ILLUSTRATION 9.12

A fair dice is thrown three times. If p , q and r are the numbers obtained on the dice, then find the probability that $i^p + i^q + i^r = 1$, where $i = \sqrt{-1}$.

Sol. The possible values of p , q and r are (1, 3, 4), (3, 4, 5), (4, 4, 6) and (2, 4, 4) not in order.
 $\therefore$ Number of favourable cases = $3! + 3! + 3 + 3 = 18$
 Total number of cases = $6 \times 6 \times 6$
 $\therefore$ Required probability = $\frac{18}{216} = \frac{1}{12}$

ILLUSTRATION 9.13

A mapping is selected at random from the set of all the mappings of the set $A = \{1, 2, \dots, n\}$ into itself. Find the probability that the mapping selected is an injection.

Sol. The mapping is from set A to A . So, domain and codomain of function are A .
 Now, each pre-image in set A can be assigned any one of the images from set A .
 So, total number mappings = $n \times n \times n \times \dots \times n$ (n times) = n^n
 Number of mappings which are one-one
 $= n \times (n-1) \times (n-2) \times \dots \times 2 \times 1 = n!$
 So, required probability = $\frac{n!}{n^n} = \frac{(n-1)!}{n^{n-1}}$

ILLUSTRATION 9.14

Two integers x and y are chosen with replacement out of the set $\{0, 1, 2, 3, \dots, 10\}$. Then find the probability that $|x - y| > 5$.

Sol. Since x and y each can take values from 0 to 10, so the total number of ways of selecting x and y is $11 \times 11 = 121$. Now,
 $|x - y| > 5$
 $\Rightarrow x - y < -5$ or $x - y > 5$
 When $x - y > 5$, we have following cases:

Value of x	Value of y	Number of cases
6	0	1
7	0, 1	2
8	0, 1, 2	3
9	0, 1, 2, 3	4
10	0, 1, 2, 3, 4	5
	Total number of cases	15

Similarly, we have 15 cases for $x - y < -5$. There are 30 pairs of values of x and y satisfying these two inequalities. So, favorable number of ways is 30. Hence, required probability is $30/121$.

ILLUSTRATION 9.15

Find the probability that the 3 N's come consecutively in the arrangement of the letters of the word "CONSTANTINOPLE".

Sol. The total number of arrangement of the letters of the word "CONSTANTINOPLE" is $(14)!/3! 2! 2!$
 Since 3 N's are consecutive, then considering all the 3 N's as single letter, the total number of arrangements is $(12)!/2! 2!$
 Therefore, required probability is

$$= \frac{(12)!/(2! 2!)}{(14)!/(3! 2! 2!)} = \frac{3!}{14 \times 13} = \frac{3}{91}$$

ILLUSTRATION 9.16

Out of $3n$ consecutive integers, three are selected at random. Find the probability that their sum is divisible by 3.

Sol. Out of $3n$ consecutive integers, n integers will be of $3k$ type, n integers will be of $3k + 1$ type and n integers will be of $3k + 2$ type.

If sum of three integers selected is divisible by 3, then we have following possibilities:

All integers are of same type.

$$\therefore \text{Number of ways of selection} = {}^nC_3 + {}^nC_3 + {}^nC_3 = 3 \times {}^nC_3$$

One integer of each type.

$$\therefore \text{Number of ways of selection} = {}^nC_1 \times {}^nC_1 \times {}^nC_1 = n^3$$

$$\text{So, number of favourable ways} = 3 \times {}^nC_3 + n^3$$

$$\text{Total number of ways} = {}^{3n}C_3$$

$$\therefore \text{Required probability} = \frac{3 \times {}^nC_3 + n^3}{{}^{3n}C_3} = \frac{3n^2 - 3n + 2}{(3n-1)(3n-2)}$$

ILLUSTRATION 9.17

Find the probability that a randomly chosen three-digit number has exactly three factors.

Sol. Number of three digits numbers = 900

If a number has exactly three factors, then it must be square of a prime number.

Squares of 11, 13, 17, 19, 23, 29, 31 are three-digit numbers.

$$\text{Therefore, required probability} = \frac{7}{900}$$

ILLUSTRATION 9.18

If p and q are chosen randomly from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ with replacement, then determine the probability that the roots of the equation $x^2 + px + q = 0$ are real.

Sol. The roots of $x^2 + px + q = 0$ will be real if $p^2 - 4q \geq 0$ or $p^2 \geq 4q$.

The possible values of p and q are given in the following table:

p	q	Number of pairs of (p, q)
2	1	1
3	1, 2	2
4	1, 2, 3, 4	4
5	1 to 6	6
6	1 to 9	9
7	1 to 10	10
8	1 to 10	10
9	1 to 10	10
10	1 to 10	10
Total		62

Also, the total number of possible pairs $(p, q) = 10 \times 10 = 100$

$$\therefore \text{Required probability} = \frac{62}{100} = 0.62$$

ILLUSTRATION 9.19

An integer is chosen at random and squared. Find the probability that the unit digit of the square is 1 or 5.

Sol. The unit digit of square will be 1 or 5 only when the unit digit of taken number is 1, 5 or 9.

$$\therefore \text{Required probability} = \frac{3}{10}$$

ILLUSTRATION 9.20

Four fair dice are thrown simultaneously. Find the probability that the highest number obtained is 4.

Sol. 4 must be obtained on at least one dice.

Let x be the number of dice on which 4 is obtained.

The remaining dice will show number less than 4 which are 1, 2 and 3.

$\therefore$ Required probability

$$= \frac{{}^4C_1 \times 3^3}{6^4} + \frac{{}^4C_2 \times 3^2}{6^4} + \frac{{}^4C_3 \times 3}{6^4} + \frac{1}{6^4} = \frac{175}{1296}$$

ILLUSTRATION 9.21

An unbiased dice, with faces numbered 1, 2, 3, 4, 5, 6, is thrown n times and the list of n numbers shown up is noted. Then find the probability that among the numbers 1, 2, 3, 4, 5, 6 only three numbers appear in this list and each number appears at least once.

Sol. When a dice is rolled n times, total number of cases is 6^n

Let us first select three numbers which appear.

Three numbers can be selected from six 6C_3 ways.

Now each of these three numbers appear at least once.

This is same as filling three different boxes with n different objects such that no box remains empty.

Number of ways of such distribution using the principle of

$$\begin{aligned} \text{inclusion-exclusion} &= 3^n - {}^3C_1(3-1)^n + {}^3C_2(3-2)^n \\ &= 3^n - 3 \times 2^n + 3 \end{aligned}$$

$$\therefore \text{Required probability} = \frac{(3^n - 3 \times 2^n + 3) \times {}^6C_3}{6^n}$$

ILLUSTRATION 9.22

Six points are there on a circle from which two triangles are drawn with no vertex common. Find the probability that none of the sides of the triangles intersect.

Sol. Let the points on the circle be A_1, A_2, A_3, A_4, A_5 and A_6 in order.

Since two triangles are drawn with no vertex common and none of the sides of the triangles intersect, three vertices of each triangle are consecutive.

One such pair of triangles is $\Delta A_1A_2A_3$ and $\Delta A_4A_5A_6$.

Similarly, we have pairs $(\Delta A_2A_3A_4, \Delta A_1A_5A_6), \dots, (\Delta A_6A_1A_2, \Delta A_3A_4A_5)$.

So, there are six favourable cases.

Also, total number of cases = 6C_3

$$\therefore \text{Required probability} = \frac{6}{{}^6C_3} = \frac{3}{10}$$

ILLUSTRATION 9.23

Balls are drawn one-by-one without replacement from a box containing 2 black, 4 white and 3 red balls till all the balls are drawn. Find the probability that the balls drawn are in the order 2 black, 4 white and 3 red.

Sol. To draw 2 black, 4 white, and 3 red balls in order is same as arranging two black balls at first 2 places, 4 white balls at next 4 places, (3rd to 6th place) and 3 red balls at next 3 places (7th to 9th place), i.e., $B_1B_2W_1W_2W_3W_4R_1R_2R_3$, which can be done in $2! \times 4! \times 3!$ ways. And total number of ways of arranging all $2 + 4 + 3 = 9$ balls is $9!$. Therefore the required probability is

$$\frac{2! \times 4! \times 3!}{9!} = \frac{1}{1260}$$

ILLUSTRATION 9.24

In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats?

Sol. We have 14 seats in two vans and there are 9 boys and 3 girls. The number of ways of arranging 12 people on 14 seats without restriction is

$${}^{14}P_{12} = \frac{14!}{2!} = 7(13!)$$

Now, the number of ways of choosing a van is 2. The number of ways of arranging girls on three adjacent seats in selected van is $2(3!)$. The number of ways of arranging 9 boys on the remaining 11 seats is ${}^{11}P_9$. Therefore, the required number of ways is

$$2 \times (2 \times 3!) \times {}^{11}P_9 = \frac{4!11!}{2!} = 12!$$

Hence, the probability of the required events is $\frac{12!}{7 \times 13!} = \frac{1}{91}$

ILLUSTRATION 9.25

Find the probability that the birthdays of six different persons will fall in exactly two calendar months.

Sol. Since anyone's birthday can fall in one of 12 months, so total number of ways is 12^6 .

Now, any two months can be chosen in ${}^{12}C_2$ ways. The six birthdays can fall in these two months in 2^6 ways. Out of these 2^6 ways there are two ways when all the six birthdays fall in one month. So, favorable number of ways is ${}^{12}C_2 \times (2^6 - 2)$. Hence, required probability is

$$\frac{{}^{12}C_2 \times (2^6 - 2)}{12^6} = \frac{12 \times 11 \times (2^5 - 1)}{12^6} = \frac{341}{12^5}$$

ILLUSTRATION 9.26

If ten objects are distributed at random among ten persons, then find the probability that at least one of them will not get any object.

Sol. Each of the 10 objects can be given to any one of the 10 persons.

So, total number of ways = 10^{10}

The number of ways of distribution in which each one gets exactly one object is $10!$.

So, the number of ways of distribution in which at least one of them does not get any object is $(10^{10} - 10!)$.

$$\therefore \text{Required probability} = \frac{10^{10} - 10!}{10^{10}}$$

ILLUSTRATION 9.27

2^n players of equal strength are playing a knock out tournament. If they are paired at randomly in all rounds, find out the probability that out of two particular players S_1 and S_2 , exactly one will reach in semi-final ($n \in N, n \geq 2$).

Sol. Four players will reach in semi-final.

Since all the players are of equal strength, this is equivalent to selecting four players out of 2^n players.

Total number of ways = ${}^{2^n}C_4$

Favorable number of ways

= Selecting 4 players from 2^n players of which 3 players are from $(2^n - 2)$ players (other than S_1 and S_2) and 1 from S_1 and S_2

$$= {}^{2^n-2}C_3 \times {}^2C_1$$

$$\begin{aligned} \therefore \text{Required probability} &= \frac{{}^{(2^n-2)}C_3 \times 2}{{}^{2^n}C_4} \\ &= \frac{(2^n - 2) \times (2^n - 3) \times (2^n - 4) \times 8}{2^n(2^n - 1)(2^n - 2)(2^n - 3)} \\ &= \frac{8 \times (2^n - 4)}{2^n(2^n - 1)} \end{aligned}$$

ILLUSTRATION 9.28

Fourteen numbered balls (1, 2, 3, ..., 14) are divided in 3 groups randomly. Find the probability that the sum of the numbers on the balls, in each group, is odd.

Sol. Each group should have odd number of odd numbered balls.

Case I: Two groups have three odd numbered balls and third has only one odd numbered ball.

$$\text{Number of such cases} = \frac{7!}{(3!)^2 \times 1! \times 2!} \times 3^7$$

(as even number of balls can go in any group)

Case II: Two groups have one odd numbered ball each and the third group has five odd numbered balls.

$$\text{Number of such cases} = \frac{7!}{(1!)^2 \times 5! \times 2!} \times 3^7$$

Total number of cases = Number of ways in which three non-empty groups can be formed

$$= \frac{3^{14} - {}^3C_1 2^{14} + {}^3C_2}{3!}$$

$$\therefore \text{Required probability} = \frac{\frac{7!}{(3!)^2} + \frac{7!}{5! \times 2!}}{3^{14} - {}^3C_1 2^{14} + {}^3C_2} \times 3! \times 3^7$$

ILLUSTRATION 9.29

Five different digits from the set of numbers $\{1, 2, 3, 4, 5, 6, 7\}$ are written in random order. Find the probability that five-digit number thus formed is divisible by 9.

Sol. If the sum of the digits in a number is divisible by 9, then the number itself is divisible by 9.

The least and the greatest sum of five digits chosen from the set are 15 and 25, respectively.

So, either number comprises $\{1, 2, 4, 5, 6\}$ or $\{1, 2, 3, 5, 7\}$.

$$\therefore \text{Number of favourable cases} = 2 \times 5!$$

$$\text{Total number of cases} = {}^7P_5$$

$$\therefore \text{Required probability} = \frac{2 \times 5!}{{}^7P_5} = \frac{2}{21}$$

ILLUSTRATION 9.30

Three married couples sit in a row. Find the probability that no husband sits with his wife.

Sol. Total number of cases = $6! = 720$

Number of favourable cases

= Total number of cases – Cases when at least one husband sits with his wife

$$= 6! - ({}^3C_1 \times 2! \times 5! - {}^3C_2 \times 2! \times 2! \times 4! + {}^3C_3 \times 2! \times 2! \times 2! \times 3!)$$

(Using principle of Inclusion–Exclusion)

$$= 720 - (720 - 288 + 48) = 240$$

$$\therefore \text{Required probability} = \frac{240}{720} = \frac{1}{3}$$

CONCEPT APPLICATION EXERCISE 9.2

1. If two fair dices are thrown and digits on dices are a and b , then find the probability for which $\omega^{ab} = 1$, (where ω is a cube root of unity).
2. There are n letters and n addressed envelopes. Find the probability that all the letters are not kept in the right envelope.
3. Find the probability of getting a total of 5 or 6 in a single throw of two dice.
4. Two integers are chosen at random and multiplied. Find the probability that the product is an even integer.
5. If out of 20 consecutive whole numbers two are chosen at random, then find the probability that their sum is odd.
6. A bag contains 3 red, 7 white, and 4 black balls. If three balls are drawn from the bag, then find the probability that all of them are of the same color.
7. An ordinary cube has four blank faces, one face marked 2 and one face marked 3. Then find the probability of obtaining a total of exactly 12 in 5 throws.
8. If the letters of the word "REGULATIONS" be arranged at random, find the probability that there will be exactly four letters between the R and the E .
9. A five-digit number is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number formed is divisible by 4.
10. Five persons entered the lift cabin on the ground floor of an eight-floor house. Suppose that each of them, independently and with equal probability, can leave the cabin at any floor beginning with the first. Find out the probability of all five persons leaving at different floors.
11. Two friends A and B have equal number of daughters. There are three cinema tickets which are to be distributed among the daughters of A and B . The probability that all the tickets go to the daughters of A is $1/20$. Find the number of daughters each of them have.
12. A bag contains 12 pairs of socks. Four socks are picked up at random. Find the probability that there is at least one pair.
13. There are eight girls among whom two are sisters, all of them are to sit on a round table. Find the probability that the two sisters do not sit together.
14. A bag contains 50 tickets numbered 1, 2, 3, ..., 50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). Find the probability that $x_3 = 30$.
15. A pack of 52 cards is divided at random into two equal parts. Find the probability that both parts will have an equal number of black and red cards.
16. Let the nine different letters $A, B, C, \dots, I \in \{1, 2, 3, \dots, 9\}$. Then find the probability that product $(A-1)(B-1) \dots (I-9)$ is an even number.
17. If two distinct numbers m and n are chosen at random from the set $\{1, 2, 3, \dots, 100\}$, then find the probability that $2^m + 2^n + 1$ is divisible by 3.

18. Two numbers a and b are chosen at random from the set of first 30 natural numbers. Find the probability that $a^2 - b^2$ is divisible by 3.
19. Twelve balls are distributed among three boxes. Find the probability that the first box will contain three balls.

ANSWERS

1. $\frac{5}{9}$
2. $1 - \frac{1}{n!}$
3. $\frac{1}{4}$
4. $\frac{2}{3}$
5. $\frac{10}{19}$
6. $\frac{10}{91}$
7. $\frac{5}{2592}$
8. $\frac{11!}{{}^9P_4 \times 6! \times 2!}$
9. $\frac{1}{5}$
10. $\frac{{}^7P_5}{7^5}$
11. 3
12. $1 - \frac{{}^{12}C_4 \times 2^4}{{}^{24}C_4}$
13. $\frac{5}{7}$
14. $\frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$
15. $\frac{\{26!\}^4}{\{13!\}^4 \times 52!}$
16. 1
17. $\frac{49}{198}$
18. $\frac{47}{87}$
19. ${}^{12}C_3 \times \frac{2^9}{3^{12}}$

ADDITION THEOREM OF PROBABILITY

If A and B are any two events in a sample space S , then the probability of occurrence of at least one of the events A and B is given by $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Proof:

From the set theory, we know that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Dividing both sides by $n(S)$, we get

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$\text{or } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Note:

- If A and B are mutually exclusive events, then $A \cap B = \phi$ and hence $P(A \cap B) = 0$.
 $\therefore P(A \cup B) = P(A) + P(B)$
- Two events A and B are mutually exclusive if and only if $P(A \cup B) = P(A) + P(B)$.
- $1 = P(S) = P(A \cup A') = P(A) + P(A')$ [$\because A \cap A' = \phi$]
or $P(A') = 1 - P(A)$

If A, B , and C are any three events in a sample space S , then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$- P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

- If A, B, C are mutually exclusive events, then
 $A \cap B = \phi, B \cap C = \phi, A \cap C = \phi, A \cap B \cap C = \phi$
 $\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C)$
- If A and B are any two events, then
 $(A - B) \cap (A \cap B) = \phi$
and $A = (A - B) \cup (A \cap B)$

$$\begin{aligned}\therefore P(A) &= P(A - B) + P(A \cap B) \\ &= P(A \cap B') + P(A \cap B) \\ \text{or } P(A) - P(A \cap B) &= P(A - B) = P(A \cap B') \\ &\quad [\because A - B = A \cap B'] \\ \text{Similarly, } P(B) - P(A \cap B) &= P(B - A) = P(B \cap A')\end{aligned}$$

$$\begin{aligned}\Rightarrow P(A \cup B) &= P(\bar{A} \cap B) + P(A) \\ \text{or } \frac{3}{4} &= P(\bar{A} \cap B) + 1 - \frac{2}{3} \\ \text{or } P(\bar{A} \cap B) &= \frac{5}{12}\end{aligned}$$

GENERAL FORM OF ADDITION THEOREM OF PROBABILITY

$$\begin{aligned}P(A_1 \cup A_2 \cup \dots \cup A_n) &= \sum_{i=1}^n P(A_i) - \sum_{i < j} P(A_i \cap A_j) \\ &\quad + \sum_{i < j < k} P(A_i \cap A_j \cap A_k) - \dots + (-1)^{n-1} P(A_1 \cap A_2 \cap \dots \cap A_n)\end{aligned}$$

Note:

For any number of mutually exclusive events $E_1, E_2, E_3, \dots, E_n$,
 $P(E_1 \cup E_2 \cup \dots \cup E_n) = P(E_1) + P(E_2) + \dots + P(E_n)$

ILLUSTRATION 9.31

A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, then find the probability that it is rusted or is a nail.

Sol. The total number of nails and nuts is $6 + 10 = 16$.

$$P(R) = \frac{1}{2} \quad (R \text{ stands for rusted})$$

$$P(N) = \frac{6}{16} \quad (N \text{ stands for nails})$$

$$P(R \cap N) = \frac{3}{16} \quad [\because 3 \text{ nails are rusted out of 6 nails}]$$

$$\begin{aligned}\therefore P(R \cup N) &= P(R) + P(N) - P(R \cap N) \\ &= \frac{1}{2} + \frac{6}{16} - \frac{3}{16} = \frac{8+6-3}{16} = \frac{11}{16}\end{aligned}$$

ILLUSTRATION 9.32

The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$.

Sol. It is given that $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$. Therefore,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\text{or } 0.6 = P(A) + P(B) - 0.2$$

$$\text{or } P(A) + P(B) = 0.8$$

$$\text{or } 1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.8$$

$$\text{or } P(\bar{A}) + P(\bar{B}) = 1.2$$

ILLUSTRATION 9.33

If $P(A \cup B) = 3/4$ and $P(\bar{A}) = 2/3$, then find the value of $P(\bar{A} \cap B)$.

Sol. Since $\bar{A} \cap B$ and A are mutually exclusive events such that
 $A \cup B = (\bar{A} \cap B) \cup A$

ILLUSTRATION 9.34

Let A, B, C be three events. If the probability of occurring exactly one event out of A and B is $1 - a$, out of B and C is $1 - 2a$, out of C and A is $1 - a$ and that of occurring three events simultaneously is a^2 , then prove that probability that at least one out of A, B, C will occur is greater than $1/2$.

Sol. $P(\text{exactly one event out of } A \text{ and } B \text{ occurs})$

$$= P[(A \cap B') \cup (A' \cap B)]$$

$$= P(A \cup B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B)$$

$$\therefore P(A) + P(B) - 2P(A \cap B) = 1 - a \quad (1)$$

Similarly,

$$P(B) + P(C) - 2P(B \cap C) = 1 - 2a \quad (2)$$

$$P(C) + P(A) - 2P(C \cap A) = 1 - a \quad (3)$$

$$P(A \cap B \cap C) = a^2 \quad (4)$$

Now,

$$\begin{aligned}P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) \\ &\quad - P(C \cap A) + P(A \cap B \cap C)\end{aligned}$$

$$\begin{aligned}&= \frac{1}{2} [P(A) + P(B) - 2P(B \cap C) + P(B) + P(C) - 2P(B \cap C) \\ &\quad + P(C) + P(A) - 2P(C \cap A)] + P(A \cap B \cap C)\end{aligned}$$

$$\begin{aligned}&= \frac{1}{2} [1 - a + 1 - 2a + 1 - a] + a^2 \\ &\quad [\text{Using Eqs. (1), (2), (3), and (4)}]\end{aligned}$$

$$= \frac{3}{2} - 2a + a^2$$

$$= \frac{1}{2} + (a - 1)^2 > \frac{1}{2} \quad (\because a \neq 1)$$

ILLUSTRATION 9.35

Let A and B be any two events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Then find the value of $P(A' \cap B')' + P(A' \cup B')'$.

Sol. $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$

$$P(A' \cup B')' = P(A \cap B)$$

and $P(A' \cap B')' = P(A \cup B)$

$$P(A' \cap B')' + P(A' \cup B')'$$

$$= P(A \cup B) + P(A \cap B)$$

$$= P(A) + P(B)$$

$$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

ILLUSTRATION 9.36

If A and B are events such that $P(A' \cup B') = \frac{3}{4}$,
 $P(A' \cap B') = \frac{1}{4}$ and $P(A) = \frac{1}{3}$, then find the value of
 $P(A' \cap B)$.

Sol. $P(A' \cup B') = \frac{3}{4}$

$$\Rightarrow P((A \cap B)') = \frac{3}{4}$$

$$\Rightarrow P(A \cap B) = \frac{1}{4}$$

$$P(A' \cap B') = \frac{1}{4}$$

$$\Rightarrow P((A \cup B)') = \frac{1}{4}$$

$$\Rightarrow P(A \cup B) = \frac{3}{4}$$

$$\text{Now, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4}$$

$$\Rightarrow P(B) = \frac{3}{4} - \frac{1}{12} = \frac{2}{3}$$

$$\therefore P(A' \cap B) = P(B) - P(A \cap B) \\ = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

ILLUSTRATION 9.37

A sample space consists of 9 elementary outcomes $E_1, E_2, \dots, E_9$ whose probabilities are:

$$P(E_1) = P(E_2) = 0.09, P(E_3) = P(E_4) = P(E_5) = 0.1, \\ P(E_6) = P(E_7) = 0.2, P(E_8) = P(E_9) = 0.06$$

If $A = \{E_1, E_5, E_8\}$, $B = \{E_2, E_5, E_8, E_9\}$, then

(a) Calculate $P(A)$, $P(B)$, and $P(A \cap B)$.

(b) Using the addition law of probability, calculate $P(A \cup B)$.

(c) List the composition of the event $A \cup B$, and calculate $P(A \cup B)$ by adding the probabilities of the elementary outcomes.

(d) Calculate $P(\bar{B})$ from $P(B)$, also calculate $P(\bar{B})$ directly from the elementary outcomes of $\bar{B}$.

Sol.

$$(a) P(A) = P(E_1) + P(E_5) + P(E_8) \\ = 0.09 + 0.1 + 0.06 = 0.25$$

$$(b) P(B) = P(E_2) + P(E_5) + P(E_8) + P(E_9) \\ = 0.09 + 0.1 + 0.06 + 0.06 = 0.31$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\text{Now, } A \cap B = \{E_5, E_8\}$$

$$\therefore P(A \cap B) = P(E_5) + P(E_8) = 0.1 + 0.06 = 0.16$$

$$\therefore P(A \cup B) = 0.25 + 0.31 - 0.16 = 0.40$$

$$(c) A \cup B = \{E_1, E_2, E_5, E_8, E_9\}$$

$$P(A \cup B) = P(E_1) + P(E_2) + P(E_5) + P(E_8) + P(E_9) \\ = 0.09 + 0.09 + 0.1 + 0.06 + 0.06 = 0.40$$

$$(d) \therefore P(\bar{B}) = 1 - P(B) = 1 - 0.31 = 0.69$$

$$\text{and } \bar{B} = \{E_1, E_3, E_4, E_6, E_7\}$$

$$\therefore P(\bar{B}) = P(E_1) + P(E_3) + P(E_4) + P(E_6) + P(E_7) \\ = 0.09 + 0.1 + 0.1 + 0.2 + 0.2 = 0.69$$

ILLUSTRATION 9.38

The following Venn diagram shows three events, A , B , and C , and also the probabilities of the various intersections.

Determine

(a) $P(A)$

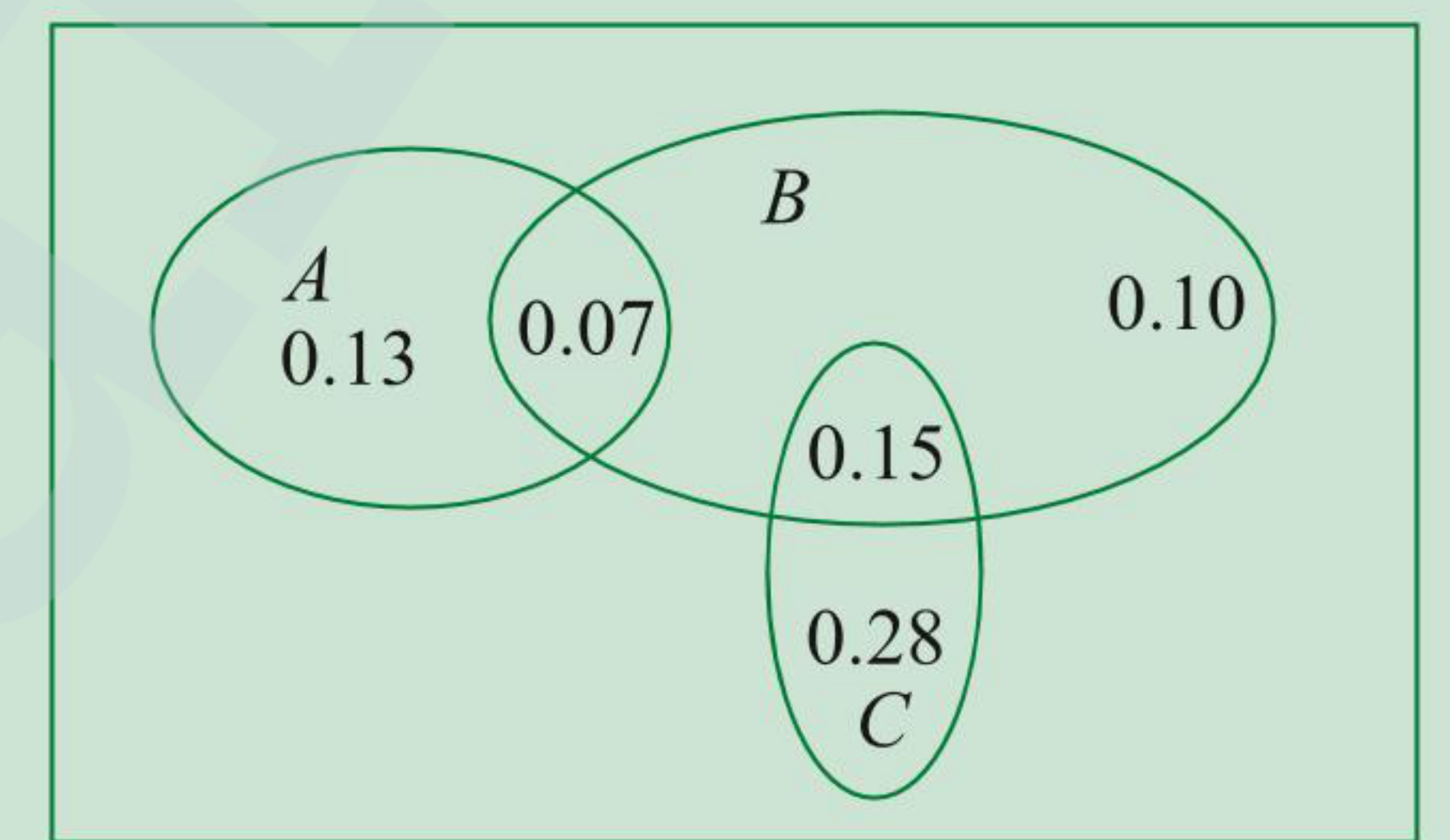
(b) $P(B \cap \bar{C})$

(c) $P(A \cup B)$

(d) $P(A \cap \bar{B})$

(e) $P(B \cap C)$

(f) Probability of the event that exactly one of A , B , and C occurs.



Sol. From the above Venn diagram,

$$(a) P(A) = 0.13 + 0.07 = 0.20$$

$$(b) P(B \cap \bar{C}) = P(B) - P(B \cap C) \\ = 0.07 + 0.10 + 0.15 - 0.15 = 0.17$$

$$(c) P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ = 0.13 + 0.07 + 0.07 + 0.10 + 0.15 - 0.07 \\ = 0.45$$

$$(d) P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.13 + 0.07 - 0.07 = 0.13$$

$$(e) P(B \cap C) = 0.15$$

$$(f) P(\text{exactly one of the three occurs}) = 0.13 + 0.10 + 0.28 \\ = 0.51$$

ILLUSTRATION 9.39

Three numbers are chosen at random without replacement from $\{1, 2, 3, \dots, 10\}$. Find the probability that the smallest of the chosen numbers is 3, or the greatest one is 7.

Sol. Let E_1 be the event “getting smallest number 3”

and E_2 be the event “getting greatest number 7”

Then, $P(E_1) = P(\text{getting one number 3 and other two from numbers 4 to 10})$

$$= \frac{{}^1C_1 \times {}^7C_2}{{}^{10}C_3} = \frac{7}{40}$$

Similarly,

$P(E_2) = P(\text{getting one number 7 and other 2 from numbers 1 to 6})$

$$= \frac{{}^1C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{1}{8}$$

$P(E_1 \cap E_2) = P(\text{getting one number 3 second number 7 and third from number from 4 to 6})$

$$= \frac{{}^1C_1 \times {}^1C_1 \times {}^3C_1}{{}^{10}C_3} = \frac{1}{40}$$

$$\begin{aligned} \therefore P(E_1 \cup E_2) &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \\ &= \frac{7}{40} + \frac{1}{8} - \frac{1}{40} \\ &= \frac{7+5-1}{40} = \frac{11}{40} \end{aligned}$$

ILLUSTRATION 9.40

If A and B are two events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$, then show that

$$\begin{aligned} \text{(a) } P(A \cup B) &\geq \frac{2}{3} & \text{(b) } \frac{1}{6} &\leq P(A \cap B) \leq \frac{1}{2} \\ \text{(c) } P(A \cap \bar{B}) &\leq \frac{1}{3} & \text{(d) } \frac{1}{6} &\leq P(\bar{A} \cap B) \leq \frac{1}{2} \end{aligned}$$

Sol. $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$

$$P(A \cup B) \geq \max \{P(A), P(B)\} = \frac{2}{3}$$

$$\text{and } P(A \cap B) \leq \min \{P(A), P(B)\} = \frac{1}{2}$$

$$\begin{aligned} \text{Now, } P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &\geq P(A) + P(B) - 1 = \frac{1}{6} \end{aligned}$$

$$\therefore \frac{1}{6} \leq P(A \cap B) \leq \frac{1}{2} \quad (1)$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B) = \frac{1}{2} - P(A \cap B)$$

$$\text{From (1), } -\frac{1}{2} \leq -P(A \cap B) \leq -\frac{1}{6}$$

$$\therefore 0 \leq \frac{1}{2} - P(A \cap B) \leq \frac{1}{2} - \frac{1}{6}$$

$$\therefore P(A \cap \bar{B}) \leq \frac{1}{3}$$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - P(A \cap B)$$

$$\therefore \frac{2}{3} - \frac{1}{2} \leq \frac{2}{3} - P(A \cap B) \leq \frac{2}{3} - \frac{1}{6}$$

$$\frac{1}{6} \leq P(\bar{A} \cap B) \leq \frac{1}{2}$$

ILLUSTRATION 9.41

Given two events A and B . If odds against A are as 2:1 and those in favor of $A \cup B$ are as 3:1, then find the range of $P(B)$.

Sol. Clearly $P(A) = 1/3$, $P(A \cup B) = 3/4$. Now,

$$P(B) \leq P(A \cup B)$$

$$\Rightarrow P(B) \leq 3/4 \quad (1)$$

$$\text{Also, } P(B) = P(A \cup B) - P(A) + P(A \cap B) \geq \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$$

$$\text{Hence, } \frac{5}{12} \leq P(B) \leq \frac{3}{4}.$$

ILLUSTRATION 9.42

The probabilities of three events A , B , and C are $P(A) = 0.6$, $P(B) = 0.4$, and $P(C) = 0.5$. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, and $P(A \cup B \cup C) \geq 0.85$, then find the range of $P(B \cap C)$.

Sol. We have,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.4 - 0.8 = 0.2$$

$$\begin{aligned} \text{Also, } P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap C) \\ &\quad + P(A \cap B \cap C) - P(A \cap B) - P(B \cap C) \end{aligned}$$

$$\Rightarrow P(B \cap C) = 1.2 - P(A \cup B \cup C) \quad (1)$$

$$\text{Now, } 0.85 \leq P(A \cup B \cup C) \leq 1$$

$$\text{From Eq. (1), } 0.2 \leq P(B \cap C) \leq 0.35$$

CONCEPT APPLICATION EXERCISE 9.3

- A and B are two candidates seeking admission to IIT. The probability that A is selected is 0.5 and the probability that A and B are selected is at most 0.3. Is it possible that the probability of B getting selected is 0.9?
- If A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, and $P(A^c) = 2/3$, then find
 - $P(A)$
 - $P(B)$
 - $P(A \cap B^c)$
 - $P(A^c \cap B)$
- If $P(A \cap B) = \frac{1}{2}$, $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$, $P(A) = p$, $P(B) = 2p$, then find the value of p .
- In a class of 125 students 70 passed in Mathematics, 55 in Statistics, and 30 in both. Then find the probability that a student selected at random from the class has passed in only one subject.
- In a certain population, 10% of the people are rich, 5% are famous, and 3% are rich and famous. Then find the probability that a person picked at random from the population is either famous or rich but not both.
- Three students A and B and C are in a swimming race. A and B have the same probability of winning and each is twice as likely to win as C . Find the probability that B or C wins. Assume no two reach the winning point simultaneously.
- Let A , B , C be three events such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$, then show that $0.23 \leq P(B \cap C) \leq 0.48$.

ANSWERS

- Not possible
- $P(A) = 1/3$; $P(B) = 2/3$; $P(A \cap B^c) = 1/12$; $P(A^c \cap B) = 5/12$
- 7/18 4. 13/25 5. 0.09 6. 3/5

Exercises

Single Correct Answer Type

- A sample space consists of 3 sample points with associated probabilities given as $2p, p^2, 4p - 1$. Then the value of p is
 (1) $p = \sqrt{11} - 3$ (2) $\sqrt{10} - 3$
 (3) $\frac{1}{4} < p < \frac{1}{2}$ (4) none of these
- Let E be an event which is neither a certainty nor an impossibility. If probability is such that $P(E) = 1 + \lambda + \lambda^2$ and $P(E') = (1 + \lambda)^2$ in terms of an unknown λ . Then $P(E)$ is equal to
 (1) 1 (2) $\frac{3}{4}$
 (3) $\frac{1}{4}$ (4) none of these
- Three balls marked with 1, 2 and 3 are placed in an urn. One ball is drawn, its number is noted, then the ball is returned to the urn. This process is repeated and then repeated once more. Each ball is equally likely to be drawn on each occasion. If the sum of the numbers noted is 6, then the probability that the ball numbered with 2 is drawn at all the three occasions, is
 (1) $\frac{1}{27}$ (2) $\frac{1}{7}$ (3) $\frac{1}{6}$ (4) $\frac{1}{3}$
- A draws a card from a pack of n cards marked 1, 2, ..., n . The card is replaced in the pack and B draws a card. Then the probability that A draws a higher card than B is
 (1) $(n + 1)/2n$ (2) $1/2$
 (3) $(n - 1)/2n$ (4) none of these
- South African cricket captain lost the toss of a coin 13 times out of 14. The chance of this happening was
 (1) $7/2^{13}$ (2) $1/2^{13}$ (3) $13/2^{14}$ (4) $13/2^{13}$
- The probability that in a family of five members, exactly two members have birthday on Sunday is
 (1) $(12 \times 5^3)/7^5$ (2) $(10 \times 6^2)/7^5$
 (3) $2/5$ (4) $(10 \times 6^3)/7^5$
- Three houses are available in a locality. Three persons apply for the houses. Each applies for one houses without consulting others. The probability that all three apply for the same houses is
 (1) $1/9$ (2) $2/9$ (3) $7/9$ (4) $8/9$
- The numbers 1, 2, 3, ..., n are arrange in a random order. The probability that the digits 1, 2, 3, ..., k ($k < n$) appear as neighbors in that order is
 (1) $1/n!$ (2) $k!/n!$
 (3) $(n - k)!/n!$ (4) $(n - k + 1)!/n!$
- Words from the letters of the word PROBABILITY are formed by taking all letters at a time. The probability that both B 's are not together and both I 's are not together is
 (1) $52/55$ (2) $53/55$
 (3) $54/55$ (4) none of these
- There are only two women among 20 persons taking part in a pleasure trip. The 20 persons are divided into two groups, each group consisting of 10 persons. Then the probability that the two women will be in the same group is
 (1) $9/19$ (2) $9/38$
 (3) $9/35$ (4) none of these
- Five different games are to be distributed among 4 children randomly. The probability that each child get at least one game is
 (1) $1/4$ (2) $15/64$
 (3) $21/64$ (4) none of these
- A drawer contains 5 brown socks and 4 blue socks well mixed. A man reaches the drawer and pulls out socks at random. What is the probability that they match?
 (1) $4/9$ (2) $5/8$ (3) $5/9$ (4) $7/12$
- A four figure number is formed of the figures 1, 2, 3, 5 with no repetitions. The probability that the number is divisible by 5 is
 (1) $3/4$ (2) $1/4$
 (3) $1/8$ (4) none of these
- Twelve balls are placed in three boxes. The probability that the first box contains three balls is
 (1) $\frac{110}{9} \left(\frac{2}{3}\right)^{10}$ (2) $\frac{9}{110} \left(\frac{2}{3}\right)^{10}$
 (3) $\frac{{}^{12}C_3}{12^3} \times 2^9$ (4) $\frac{{}^{12}C_3}{3^{12}}$
- A cricket club has 15 members, of whom only 5 can bowl. If the names of 15 members are put into a box and 11 are drawn at random, then the probability of getting an eleven containing at least 3 bowlers is
 (1) $7/13$ (2) $6/13$ (3) $11/15$ (4) $12/13$
- Seven girls $G_1, G_2, G_3, \dots, G_7$ are such that their ages are in order $G_1 < G_2 < G_3 < \dots < G_7$. Five girls are selected at random and arranged in increasing order of their ages. The probability that G_5 and G_7 are not consecutive is
 (1) $\frac{20}{21}$ (2) $\frac{19}{21}$ (3) $\frac{17}{21}$ (4) $\frac{13}{21}$
- A local post office is to send M telegrams which are distributed at random over N communication channels, ($N > M$). Each telegram is sent over any channel with equal probability. Chance that not more than one telegram will be sent over each channel is
 (1) $\frac{{}^N C_M \times N!}{M^N}$ (2) $\frac{{}^N C_M \times M!}{N^M}$
 (3) $1 - \frac{{}^N C_M \times M!}{M^N}$ (4) $1 - \frac{{}^N C_M \times N!}{N^M}$

18. Dialing a telephone number an old man forgets the last two digits remembering only that these are different dialed at random. The probability that the number is dialed correctly is
 (1) $1/45$ (2) $1/90$
 (3) $1/100$ (4) none of these
19. A and B toss a fair coin each simultaneously 50 times. The probability that both of them will not get tail at the same toss is
 (1) $(3/4)^{50}$ (2) $(2/7)^{50}$ (3) $(1/8)^{50}$ (4) $(7/8)^{50}$
20. In a game called "odd man out" m ($m > 2$) persons toss a coin to determine who will buy refreshments for the entire group. A person who gets an outcome different from that of the rest of the members of the group is called the odd man out. The probability that there is a loser in any game is
 (1) $1/2m$ (2) $m/2^{m-1}$
 (3) $2/m$ (4) none of these
21. $2n$ boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is
 (1) $n/(2n-1)$ (2) $(n-1)/(2n-1)$
 (3) $(n-1)/4n^2$ (4) none of these
22. If the papers of 4 students can be checked by any one of the 7 teachers, then the probability that all the 4 papers are checked by exactly 2 teachers is
 (1) $2/7$ (2) $12/49$ (3) $32/343$ (4) $6/49$
23. If the events A and B are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$ and $P(B) = \frac{1-x}{4}$, then the set of possible real values of x lies in the interval
 (1) $[0, 1]$ (2) $\left[-\frac{1}{3}, \frac{5}{9}\right]$
 (3) $\left[-\frac{7}{9}, \frac{4}{9}\right]$ (4) $\left[\frac{1}{3}, \frac{2}{3}\right]$
24. A natural number is chosen at random from the first 100 natural numbers. The probability that $x + \frac{100}{x} > 50$ is
 (1) $1/10$ (2) $11/50$
 (3) $11/20$ (4) none of these
25. A dice is thrown six times, it being known that each time a different digit is shown. The probability that a sum of 12 will be obtained in the first three throws is
 (1) $\frac{5}{24}$ (2) $\frac{25}{216}$ (3) $\frac{3}{20}$ (4) $\frac{1}{12}$
26. If a is an integer lying in $[-5, 30]$, then the probability that the graph of $y = x^2 + 2(a+4)x - 5a + 64$ is strictly above the x -axis is
 (1) $1/6$ (2) $7/36$ (3) $2/9$ (4) $3/5$
27. Four die are thrown simultaneously. The probability that 4 and 3 appear on two of the die given that 5 and 6 have appeared on other two die is
 (1) $1/6$ (2) $1/36$
 (3) $12/151$ (4) none of these
28. A $2n$ digit number starts with 2 and all its digits are prime, then the probability that the sum of any two consecutive digits of the number is prime is
 (1) 4×2^{-3n} (2) 4×2^{-3n}
 (3) 2^{-3n} (4) none of these
29. In a n -sided regular polygon, the probability that the two diagonal chosen at random will intersect inside the polygon is
 (1) $\frac{2 {}^nC_2}{({}^nC_2 - n) C_2}$ (2) $\frac{n(n-1) C_2}{({}^nC_2 - n) C_2}$
 (3) $\frac{{}^nC_4}{({}^nC_2 - n) C_2}$ (4) none of these
30. A three-digit number is selected at random from the set of all three-digit numbers. The probability that the number selected has all the three digits same is
 (1) $1/9$ (2) $1/10$ (3) $1/50$ (4) $1/100$
31. Two numbers a, b are chosen from the set of integers 1, 2, 3, ..., 39. Then probability that the equation $7a - 9b = 0$ is satisfied is
 (1) $1/247$ (2) $2/247$ (3) $4/741$ (4) $5/741$
32. One mapping is selected at random from all mappings of the set $S = \{1, 2, 3, \dots, n\}$ into itself. If the probability that the mapping is one-one is $3/32$, then the value of n is
 (1) 2 (2) 3
 (3) 4 (4) none of these
33. A composite number is selected at random from the first 30 natural numbers and it is divided by 5. The probability that there will be a remainder is
 (1) $14/19$ (2) $5/19$ (3) $5/6$ (4) $7/15$
34. Forty teams play a tournament. Each team plays every other team just once. Each game results in a win for one team. If each team has a 50% chance of winning each game, the probability that at the end of the tournament, every team has won a different number of games is
 (1) $1/780$ (2) $40!/2^{780}$
 (3) $40!/3^{780}$ (4) none of these
35. If three squares are selected at random from chessboard, then the probability that they form the letter "L" is
 (1) $196/{}^{64}C_3$ (2) $49/{}^{64}C_3$
 (3) $36/{}^{64}C_3$ (4) $98/{}^{64}C_3$
36. A bag has 10 balls. Six balls are drawn in an attempt and replaced. Then another draw of 5 balls is made from the bag. The probability that exactly two balls are common to both the draw is
 (1) $5/21$ (2) $2/21$ (3) $7/21$ (4) $3/21$
37. The probability that a random chosen three-digit number has exactly 3 factors is
 (1) $2/225$ (2) $7/900$
 (3) $1/800$ (4) none of these

38. Let p, q be chosen one by one from the set $\{1, \sqrt{2}, \sqrt{3}, 2, e, \pi\}$ with replacement. Now a circle is drawn taking (p, q) as its centre. Then the probability that at the most two rational points exist on the circle is (rational points are those points whose both the coordinates are rational)
- (1) $2/3$ (2) $7/8$
(3) $8/9$ (4) none of these
39. Three integers are chosen at random from the set of first 20 natural numbers. The chance that their product is a multiple of 3 is
- (1) $194/285$ (2) $1/57$
(3) $13/19$ (4) $3/4$
40. Five different marbles are placed in 5 different boxes randomly. Then the probability that exactly two boxes remain empty is (each box can hold any number of marbles)
- (1) $2/5$ (2) $12/25$
(3) $3/5$ (4) none of these
41. There are 10 prizes, five A 's, three B 's, and two C 's, placed in identical sealed envelopes for the top 10 contestants in a mathematics contest. The prizes are awarded by allowing winners to select an envelope at random from those remaining. When the 8th contestant goes to select the prize, the probability that the remaining three prizes are one A , one B and one C is
- (1) $1/4$ (2) $1/3$ (3) $1/12$ (4) $1/10$
42. A car is parked among N cars standing in a row, but not at either end. On his return, the owner finds that exactly " r " of the N places are still occupied. The probability that the places neighboring his car are empty is
- (1) $\frac{(r-1)!}{(N-1)!}$ (2) $\frac{(r-1)! (N-r)!}{(N-1)!}$
(3) $\frac{(N-r)(N-r-1)}{(N+1)(N+2)}$ (4) $\frac{{}^{N-r}C_2}{{}^{N-1}C_2}$
43. Let A be a set containing n elements. A subset P of the set A is chosen at random. The set A is reconstructed by replacing the elements of P , and another subset Q of A is chosen at random. The probability that $P \cap Q$ contains exactly m ($m < n$) elements is
- (1) $\frac{3^{n-m}}{4^n}$ (2) $\frac{{}^nC_m \cdot 3^m}{4^n}$
(3) $\frac{{}^nC_m 3^{n-m}}{4^n}$ (4) None of these
44. Consider $f(x) = x^3 + ax^2 + bx + c$. Parameters a, b, c are chosen, respectively, by throwing a die three times. Then the probability that $f(x)$ is an increasing function is
- (1) $5/36$ (2) $8/36$ (3) $4/9$ (4) $1/3$
45. If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement. Then the probability that $\lim_{x \rightarrow 0} [(a^x + b^x) / 2]^{2/x} = 6$ is
- (1) $1/3$ (2) $1/4$ (3) $1/9$ (4) $2/9$

46. Mr. A lives at origin on the Cartesian plane and has his office at $(4, 5)$. His friend lives at $(2, 3)$ on the same plane. Mr. A can go to his office traveling one block at a time either in the $+y$ or $+x$ direction. If all possible paths are equally likely then the probability that Mr. A passed his friend's house is (shortest path for any event must be considered)
- (1) $1/2$ (2) $10/21$ (3) $1/4$ (4) $11/21$

Multiple Correct Answers Type

1. If A and B are two events, the probability that exactly one of them occurs is given by
- (1) $P(A) + P(B) - 2P(A \cap B)$
(2) $P(A \cap \bar{B}) + P(\bar{A} \cap B)$
(3) $P(A \cup B) - P(A \cap B)$
(4) $P(\bar{A}) + P(\bar{B}) - 2P(\bar{A} \cap \bar{B})$
2. If p and q are chosen randomly from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ with replacement, then the probability that the roots of the equation $x^2 + px + q = 0$
- (1) are real is $33/50$
(2) are imaginary is $19/50$
(3) are real and equal is $3/50$
(4) are real and distinct is $3/5$
3. If A and B are two events such that $P(A) = 3/4$ and $P(B) = 5/8$, then
- (1) $P(A \cup B) \geq 3/4$ (2) $P(A' \cap B) \leq 1/4$
(3) $1/8 \leq P(A \cap B') \leq 3/8$ (4) $3/8 \leq P(A \cap B) \leq 5/8$
4. If A and B are two mutually exclusive events, then
- (1) $P(A) \leq P(\bar{B})$ (2) $P(A) > P(B)$
(3) $P(B) \leq P(\bar{A})$ (4) $P(A) > P(B)$
5. Probability if n heads in $2n$ tosses of a fair coin can be given by
- (1) $\prod_{r=1}^n \left(\frac{2r-1}{2r} \right)$ (2) $\prod_{r=1}^n \left(\frac{n+r}{2r} \right)$
(3) $\sum_{r=0}^n \left(\frac{{}^nC_r}{2^n} \right)^2$ (4) $\frac{\sum_{r=0}^n ({}^nC_r)^2}{\left(\sum_{r=0}^{2n} {}^{2n}C_r \right)}$
6. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are
- (1) $p_1 = 1/9$ (2) $p_1 = 1/16$
(3) $p_2 = 1/3$ (4) $p_2 = 1/4$
7. A bag contains b blue balls and r red balls. If two balls are drawn at random, the probability of drawing two red balls is five times the probability of drawing two blue balls. Furthermore, the probability of drawing one ball of each

color is six times the probability of drawing two blue balls. Then

- (1) $b + r = 9$ (2) $br = 18$
 (3) $|b - r| = 4$ (4) $b/r = 2$

8. Two numbers are chosen from $\{1, 2, 3, 4, 5, 6, 7, 8\}$ one after another without replacement. Then the probability that

- (1) the smaller value of two is less than 3 is $13/28$
 (2) the bigger value of two is more than 5 is $9/14$
 (3) product of two number is even is $11/14$
 (4) none of these

Linked Comprehension Type

For Problems 1–3

A shopping mall is running a scheme: Each packet of detergent SURF contains a coupon which bears letter of the word SURF, if a person buys at least four packets of detergent SURF, and produce all the letters of the word SURF, then he gets one free packet of detergent.

- If a person buys 8 such packets at a time, then the number of different combinations of coupon he has is
 (1) 4^8 (2) 8^4 (3) ${}^{11}C_3$ (4) ${}^{12}C_4$
- If person buys 8 such packets, then the probability that he gets exactly one free packets is
 (1) $7/33$ (2) $102/495$
 (3) $13/55$ (4) $34/165$
- If a person buys 8 such packets, then the probability that he gets two free packets is
 (1) $1/7$ (2) $1/5$ (3) $1/42$ (4) $1/165$

For Problems 4–6

There are two die A and B both having six faces. Die A has three faces marked with 1, two faces marked with 2, and one face marked with 3. Die B has one face marked with 1, two faces marked with 2, and three faces marked with 3. Both dices are thrown randomly once. If E be the event of getting sum of the numbers appearing on top faces equal to x and let $P(E)$ be the probability of event E , then

- $P(E)$ is maximum when x equal to
 (1) 5 (2) 3 (3) 4 (4) 6
- $P(E)$ is minimum when x equals to
 (1) 3 (2) 4 (3) 5 (4) 6
- When $x = 4$, then $P(E)$ is equal to
 (1) $5/9$ (2) $6/7$ (3) $7/18$ (4) $8/19$

For Problems 7–9

A cube having all of its sides painted is cut by two horizontal, two vertical, and other two planes so as to form 27 cubes all having the same dimensions. Of these cubes, a cube is selected at random.

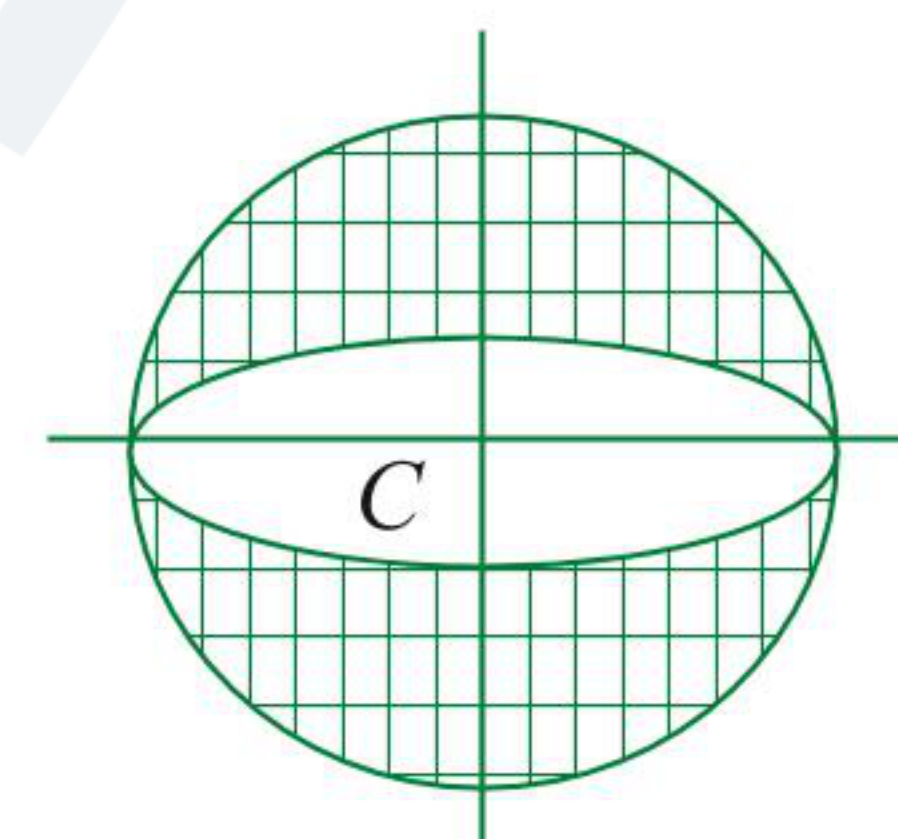
- The probability that the cube selected has none of its sides painted is
 (1) $1/9$ (2) $1/27$ (3) $1/18$ (4) $5/54$

- The probability that the cube selected has two sides painted is
 (1) $1/9$ (2) $4/9$
 (3) $8/27$ (4) none of these

- The total number of cubes having at least one of its sides painted is
 (1) 18 (2) 20 (3) 22 (4) 26

For Problems 10 and 11

There are some experiments in which the outcomes cannot be identified discretely. For example, an ellipse of eccentricity $2\sqrt{2}/3$ is inscribed in a circle and a point within the circle is chosen at random. Now, we want to find the probability that this point lies outside the ellipse. Then, the point must lie in the shaded region shown in Figure. Let the radius of the circle be a and length of minor axis of the ellipse be $2b$. Given that



$$1 - \frac{b^2}{a^2} = \frac{8}{9} \text{ or } \frac{b^2}{a^2} = \frac{1}{9}$$

Then, the area of circle serves as sample space and area of the shaded region represents the area for favorable cases. Then, required probability is

$$p = \frac{\text{Area of shaded region}}{\text{Area of circle}} = \frac{\pi a^2 - \pi ab}{\pi a^2} = 1 - \frac{b}{a} = 1 - \frac{1}{3} = \frac{2}{3}$$

Now answer the following questions.

- A point is selected at random inside a circle. The probability that the point is closer to the center of the circle than to its circumference is
 (1) $1/4$ (2) $1/2$ (3) $1/3$ (4) $1/\sqrt{2}$
- Two persons A and B agree to meet at a place between 5 and 6 pm. The first one to arrive waits for 20 min and then leave. If the time of their arrival be independent and at random, then the probability that A and B meet is
 (1) $1/3$ (2) $1/3$ (3) $2/3$ (4) $5/9$

For Problems 12–14

If the squares of a 8×8 chessboard are painted either red or black at random.

- The probability that not all the squares in any column are alternating in color is
 (1) $(1 - 1/2^7)^8$ (2) $1/2^{56}$
 (3) $1 - 1/2^7$ (4) none of these
- The probability that the chessboard contains equal number of red and black squares is
 (1) $\frac{{}^{64}C_{32}}{2^{64}}$ (2) $\frac{64!}{32! \cdot 2^{64}}$
 (3) $\frac{2^{32} - 1}{2^{64}}$ (4) none of these

14. The probability that all the squares in any column are of same color and that of a row are of alternating color is
- (1) $1/2^{64}$

(2) $1/2^{63}$

(3) $1/2$

(4) none of these

Matrix Match Type

1. n whole numbers are randomly chosen and multiplied. Now, match the following lists:

List I	List II
a. The probability that the last digit is 1, 3, 7, or 9 is	p. $\frac{8^n - 4^n}{10^n}$
b. The probability that the last digit is 2, 4, 6, 8 is	q. $\frac{5^n - 4^n}{10^n}$
c. The probability that the last digit is 5 is	r. $\frac{4^n}{10^n}$
d. The probability that the last digit is zero is	s. $\frac{10^n - 8^n - 5^n + 4^n}{10^n}$

2. Three distinct numbers a, b and c are chosen at random from the numbers 1, 2, ..., 100. The probability that

List I	List II
a. a, b, c are in AP is	p. $\frac{53}{161700}$
b. a, b, c are in GP is	q. $\frac{1}{66}$
c. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in GP is	r. $\frac{1}{22}$
d. $a + b + c$ is divisible by 2 is	s. $\frac{1}{2}$

Codes

	a	b	c	d
(1)	q	s	s	r
(2)	r	q	q	p
(3)	q	p	p	s
(4)	q	s	p	r

Numerical Value Type

1. If the probability of a six-digit number N whose six digits are 1, 2, 3, 4, 5, 6 written as random order is divisible by 6 is p , then the value of $1/p$ is _____.
2. If the probability that the product of the outcomes of three rolls of a fair dice is a prime number is p , then the value of $1/(4p)$ is _____.
3. There are two red, two blue, two white, and certain number (greater than 0) of green socks in a drawer. If two socks are taken at random from the drawer without replacement, the probability that they are of the same color is $1/5$, then the number of green socks are _____.
4. A dice is weighted such that the probability of rolling the face numbered n is proportional to n^2 ($n = 1, 2, 3, 4, 5, 6$). The dice is rolled twice, yielding the numbers a and b . The probability that $a < b$ is p then the value of $[2/p]$ (where $[\cdot]$ represents greatest integer function) is _____.
5. In a knockout tournament, 2^n equally skilled players; $S_1, S_2, \dots, S_{2^n}$ are participating. In each round, players are divided in pairs at random and winner from each pair moves to the next round. If S_2 reaches the semi-final, then the probability that S_1 wins the tournament is $\frac{1}{84}$. The value of n is _____.
6. Five different games are to be distributed among four children randomly. The probability that each child get at least one game is p , then the value of $[1/p]$ is, where $[\cdot]$ represents the greatest integer function, _____.
7. A bag contains 10 different balls. Five balls are drawn simultaneously and then replaced and then seven balls are drawn. If the probability that exactly three balls are common to the two draws is p , then the value of $8p$ is _____.

Archives

JEE ADVANCED

Single Correct Answer Type

1. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is
- (1) $1/18$

(2) $1/9$

(3) $2/9$

(4) $1/36$
- (IIT-JEE, 2010)
2. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is

- (1) $\frac{1}{2}$

(2) $\frac{1}{3}$

(3) $\frac{2}{3}$

(4) $\frac{3}{4}$
- (JEE Advanced 2014)
3. Three randomly chosen non-negative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is
- (1) $\frac{1}{2}$

(2) $\frac{36}{55}$

(3) $\frac{6}{11}$

(4) $\frac{5}{11}$
- (JEE Advanced 2017)

Multiple Correct Answers Type

1. There are three bags B_1 , B_2 and B_3 . The bag B_1 contains 5 red and 5 green balls. B_2 contains 3 red and 5 green balls and B_3 contains 5 red and 3 green balls. Bags B_1 , B_2 and B_3 have probabilities $3/10$, $3/10$ and $4/10$, respectively, of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

- (1) Probability that the chosen ball is green, given that the selected bag is B_3 , equals $\frac{3}{8}$.
 (2) Probability that the selected bag is B_3 , given that the chosen ball is green, equals $\frac{5}{13}$.
 (3) Probability that the chosen ball is green equals $\frac{39}{80}$.
 (4) Probability that the selected bag is B_3 , given that the chosen ball is green, equals $\frac{3}{10}$.

(JEE Advanced 2019)

2. Let E , F and G be three events having probabilities

$$P(E) = \frac{1}{8}, P(F) = \frac{1}{6} \text{ and } P(G) = \frac{1}{4}, \text{ and let}$$

$$P(E \cap F \cap G) = \frac{1}{10}.$$

For any event H , if H^C denotes its complement, then which of the following statements is (are) TRUE ?

- (1) $P(E \cap F \cap G^C) \leq \frac{1}{40}$ (2) $P(E^C \cap F \cap G) \leq \frac{1}{15}$
 (3) $P(E \cup F \cup G) \leq \frac{13}{24}$ (4) $P(E^C \cap F^C \cap G^C) \leq \frac{5}{12}$

(JEE Advanced 2021)**Linked Comprehension Type****For Problems 1 and 2**

Box 1 contains three cards bearing numbers 1, 2, 3; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $i = 1, 2, 3$.

(JEE Advanced 2014)

1. The probability that $x_1 + x_2 + x_3$ is odd, is

- (1) $\frac{29}{105}$ (2) $\frac{53}{105}$
 (3) $\frac{57}{105}$ (4) $\frac{1}{2}$

2. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

- (1) $\frac{9}{105}$ (2) $\frac{10}{105}$
 (3) $\frac{11}{105}$ (4) $\frac{7}{105}$

For Problems 3 and 4

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i , $i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(JEE Advanced 2018)

3. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 , and NONE of the remaining students gets the seat previously allotted to him/her, is

- (1) $\frac{3}{40}$ (2) $\frac{1}{8}$
 (3) $\frac{7}{40}$ (4) $\frac{1}{5}$

4. For $i = 1, 2, 3, 4$, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

- (1) $\frac{1}{15}$ (2) $\frac{1}{10}$
 (3) $\frac{7}{60}$ (4) $\frac{1}{5}$

Numerical Value Type**Paragraph for Questions 1 and 2:**

Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \dots, 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40

(JEE Advanced 2021)

1. The value of $\frac{625}{4} p_1$ is _____.

2. The value of $\frac{125}{4} p_2$ is _____.

3. A number is chosen at random from the set $\{1, 2, 3, \dots, 2000\}$. Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of $500p$ is _____.

(JEE Advanced 2021)

Answers Key

EXERCISES

Single Correct Answer Type

1. (1)

6. (4)

11. (2)

16. (3)

21. (1)

26. (3)

31. (3)

36. (1)

41. (1)

46. (2)
2. (2)

7. (1)

12. (1)

17. (2)

22. (4)

27. (3)

32. (3)

37. (2)

42. (4)
3. (2)

8. (4)

13. (2)

18. (2)

23. (2)

28. (2)

33. (1)

38. (3)

43. (3)
4. (3)

9. (2)

14. (1)

19. (1)

24. (3)

29. (3)

34. (2)

39. (1)

44. (3)
5. (1)

10. (1)

15. (4)

20. (2)

25. (3)

30. (4)

35. (1)

40. (3)

45. (3)

Multiple Correct Answers Type

1. (1), (2), (3), (4)

4. (1), (3)

7. (1), (2)
2. (2), (3), (4)

5. (1), (3), (4)

8. (1), (2), (3)
3. (1), (2), (3), (4)

6. (1), (3)

Linked Comprehension Type

1. (3)

6. (3)

11. (4)
2. (4)

7. (2)

12. (1)
3. (4)

8. (2)

13. (1)
4. (3)

9. (4)

14. (2)
5. (4)

10. (1)

Matrix Match Type

1. $a \rightarrow r; b \rightarrow p; c \rightarrow q; d \rightarrow s.$
2. (3)

Numerical Value Type

1. (2)

6. (4)
2. (6)

7. (7)
3. (4)
4. (5)
5. (6)

ARCHIVES

JEE ADVANCED

Single Correct Answer Type

1. (3)

2. (1)

3. (3)

Multiple Correct Answers Type

1. (1), (3)

2. (1), (2), (3)

Linked Comprehension Type

1. (2)

2. (3)

3. (1)

4. (3)

Numerical Value Type

1. (76.25)

2. (24.5)

3. (214)

Chapter 9

Concept Application Exercises

Exercise 9.1

1. (a) In assignment (a), each number is positive and less than 1
Sum of probabilities

$$= 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1$$

Thus, this assignment is valid.

- (b) In assignment (b), each number is positive and less than 1.

$$\begin{aligned}\text{Sum of probabilities} &= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} \\ &= 7 \times \frac{1}{7} = 1\end{aligned}$$

Thus, this assignment is valid.

- (c) In assignment (c), each number is positive and less than 1
Sum of probabilities

$$\begin{aligned}&= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 \\ &= 2.8 \neq 1\end{aligned}$$

Thus, the assignment is not valid.

- (d) In assignment, two numbers are negative. So, it is not valid assignment.

- (e) In assignment, $p(\omega_7) = \frac{15}{14} > 1$. So, it is not valid assignment.

2. (a) $P(X \text{ is a prime number}) = P(2) + P(3) + P(5) + P(7)$
 $= 0.23 + 0.12 + 0.20 + 0.07$
 $= 0.62$

- (b) $P(X \text{ is a number greater than 4}) = P(5) + P(6) + P(7) + P(8)$
 $= 0.20 + 0.08 + 0.07 + 0.05$
 $= 0.4$

3. There are 366 days in a leap year. Also, there are 52 weeks and 2 days.

The possible combinations of two days are as follows:

Sunday-Monday,
Monday-Tuesday,
Tuesday-Wednesday,
Wednesday-Thursday,
Thursday-Friday,
Friday-Saturday,
Saturday-Sunday

Now, let E be the event in which leap year will have 53 Fridays or 53 Saturdays.

We observe that there are three possibilities for event E, viz. Thursday-Friday, Friday-Saturday, and Saturday-Sunday.

Since each pair of 2 days is equally likely,

$$P(E) = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{3}{7}$$

4. Since the probabilities of the faces are proportional to the numbers on them, we can take the probabilities of faces 1, 2, ..., 6 as $k, 2k, \dots, 6k$, respectively. Since one of the faces must occur, we have
 $k + 2k + 3k + 4k + 5k + 6k = 1$ or $k = 1/21$

Therefore, the probability of occurrence of an even number is
 $2k + 4k + 6k = 12k = 12/21 = 4/7$.

5. There are 52 cards in the pack and each card has equal probability of getting drawn that is $\frac{1}{52}$.

There are four king and four ace cards.

So, probability of drawing either an ace or a king

$$\begin{aligned}&= \frac{1}{52} + \frac{1}{52} + \dots + \frac{1}{52} \text{ (8 times)} \\ &= \frac{8}{52} = \frac{2}{13}\end{aligned}$$

6. Sample space = $\{Y_1, Y_2, Y_3, R_1, R_2, B\}$, where Y stands for yellow, R for red and B for blue.

$$\text{Now, } P(Y_1) = P(Y_2) = P(Y_3) = P(R_1) = P(R_2) = P(B) = \frac{1}{6}.$$

$$\therefore P(Y) = P(Y_1) + P(Y_2) + P(Y_3) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

$$P(R) = P(R_1) + P(R_2) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$\text{And } P(B) = \frac{1}{6}$$

Exercise 9.2

1. Total number of cases $n(S) = 36$

Since $\omega^{ab} = 1$, ab must be multiple of 3.

So, at least one of a and b must be multiple of 3.

If none of the dices shows 3 or 6, number of cases is $4 \times 4 = 16$.

So, number of cases in which at least one of the dices shows 3 or 6 is $36 - 16 = 20$.

$$\therefore \text{Required probability} = \frac{20}{36} = \frac{5}{9}$$

2. Required probability is $1 - P(\text{all letters in right envelope})$
 $= 1 - 1/n!$

(As there are total number of $n!$ ways in which letters can take envelopes and just one way in which they have corresponding envelopes.)

3. Total number of ways is 36. Favorable cases are (1, 4), (2, 3), (3, 2), (4, 1), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1). Total number of favorable cases is 9.

Hence, the required probability is $9/36 = 1/4$.

4. If both integers are even, then product is even. If both integers are odd, then product is odd. If one integer is odd and other is even, then product is even. Therefore, the required probability is $2/3$.

5. The total number of ways in which 2 integers can be chosen from the given 20 integers is ${}^{20}C_2$.

The sum of the selected numbers is odd if exactly one of them is even and only one is odd. Therefore, the favorable number of outcomes is ${}^{10}C_1 \times {}^{10}C_1$.

Therefore, the required probability is

$$\frac{{}^{10}C_1 \times {}^{10}C_1}{{}^{20}C_2} = \frac{10}{19}$$

6. The required probability is

$$\frac{{}^3C_3 + {}^7C_3 + {}^4C_3}{{}^{14}C_3} = \frac{1 + 35 + 4}{14 \times 13 \times 2} = \frac{40}{14 \times 26} = \frac{10}{91}$$

7. The total number of ways is $n = 6^5$. A total of 12 in 5 throw can be obtained in following two ways:

- (i) one blank and four 3's can be obtained in ${}^5C_1 = 5$ ways.
(ii) three 2's and two 3's can be obtained in ${}^5C_2 = 10$ ways.

Hence, the required probability is $15/6^5 = 5/2592$.

8. Total number of ways of arranging 11 letters is $(11)!$. The number of selection of 4 letters to be placed between R and E from remaining 9 is 9C_4 . These four letters can be permuted in $4!$ ways. Now R and E can be interchanged in $2!$ ways.

$$\underbrace{(R \cdot \cdot \cdot \cdot E) \cdot \cdot \cdot \cdot \cdot}_{4 \text{ letters}} \underbrace{\cdot \cdot \cdot \cdot \cdot}_{5 \text{ letters}}$$

Hence, the number of favorable ways is ${}^9C_4 \times 4! \times 6! \times 2!$

Therefore, the required probability is

$$\frac{11!}{{}^9P_4 \times 6! \times 2!}$$

9. Total number of numbers formed by the digits 1, 2, 3, 4, 5 without repetition is $5! = 120$. Now we know that a number is divisible by 4 if the number placed at its last two digits is divisible by 4. So that last two digits can be 12, 24, 32 or 52, i.e., they can be filled in 4 ways. There are $3! = 6$ ways of filling the other three places.

Therefore, the favorable number of ways is $4 \times 6 = 24$.

Hence, the required probability is

$$P = \frac{\text{Favorable no. of cases}}{\text{Exhaustive no. of cases}} = \frac{24}{120} = \frac{1}{5}$$

10. In an 8-floor house, there are 7 floors above the ground floor.

Each person can leave the cabin at any of the seven floors, i.e., each person can leave the cabin in 7 ways. Thus, total number of ways into which 5 persons can leave the cabin is 7^5 . Now number of the ways of leaving the cabin by 5 person each at different floor is 7P_5 .

Hence, the required probability is ${}^7P_5/7^5$.

11. Let each of the two friends have n daughters. Then probability that all the tickets go to the daughters of A is

$$\frac{{}^nC_3}{{}^{2n}C_3} = \frac{n-2}{4(2n-1)} = \frac{1}{20}$$

$$\Rightarrow n = 3$$

12. The number of ways of selecting 4 socks from 24 socks is $n(S) = {}^{24}C_4$.

The number of ways of selecting 4 socks from different pairs is

$$n(E) = {}^{12}C_4 \times 2^4$$

$$\Rightarrow P(E) = \frac{{}^{12}C_4 \times 2^4}{{}^{24}C_4}$$

Hence, the probability of getting at least one pair is

$$1 - \frac{{}^{12}C_4 \times 2^4}{{}^{24}C_4}$$

13. Total number of ways is $7!$. Favorable number of ways is $7! - 2(6!)$. Hence, the probability is

$$\frac{7! - 2(6!)}{7!} = 1 - \frac{2}{7} = \frac{5}{7}$$

14. Five tickets out of 50 can be drawn in ${}^{50}C_5$ ways. Since $x_1 < x_2 < x_3 < x_4 < x_5$ and $x_3 = 30$, we have $x_1, x_2 < 30$, i.e., x_1 and x_2 should come from tickets numbered 1 and 29 and this may happen in ${}^{29}C_2$ ways. Remaining ways, i.e., $x_4, x_5 > 30$, should come from 20 tickets numbered 31 to 50 in ${}^{20}C_2$ ways.

So, favorable number of cases is ${}^{29}C_2 \cdot {}^{20}C_2$. Hence, required probability is

$$\frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$$

15. We have to find the probability that out of 26 cards drawn at random from the pack of cards, 13 will be red and 13 black. The total number of different ways to draw 26 cards from 52 is ${}^{52}C_{26}$. The favorable ways are those in which there will be 13 cards drawn from 26 red cards and 13 from 26 black cards. 13 red cards may be drawn in ${}^{26}C_{13}$ different ways and 13 black cards also in ${}^{26}C_{13}$ different ways. Therefore, total number of favorable ways is equal to ${}^{26}C_{13} \cdot {}^{26}C_{13}$. And consequently the required probability is

$$P = \frac{{}^{26}C_{13} \cdot {}^{26}C_{13}}{{}^{52}C_{26}} = \frac{\left(\frac{(26)!}{(13)!(13)!} \cdot \frac{(26)!}{(13)!(13)!} \right)}{\left(\frac{(52)!}{(26)!(26)!} \right)} = \frac{\{(26)!\}^4}{\{(13)!\}^4 (52)!}$$

16. Given set of numbers is $\{1, 2, \dots, 9\}$ in which 4 are even and 5 are odd, so in the given product it is not possible to arrange to subtract only even number from odd number, there must be at least one factor involving subtraction of an odd number from another odd number. So at least one of the factor is even. Hence, product is always even.

$\therefore$ Required probability = 1

17. $2^m + 2^n + 1 = (3-1)^m + (3-1)^n + 1$
 $= 3k + (-1)^m + (-1)^n + 1$

This is divisible by 3 if both m and n are even.

$$\therefore \text{Required probability} = \frac{{}^{50}C_2}{{}^{100}C_2} = \frac{50 \times 49}{100 \times 99} = \frac{49}{198}$$

18. The total number of ways of choosing two numbers out of 1, 2, 3, ..., 30 is ${}^{30}C_2 = 435$.

Now, 30 natural numbers are categorized as

$3k$ type: 3, 6, 9, ..., 30 $\rightarrow$ 10 numbers

$3k+1$ type: 1, 4, 7, ..., 28 $\rightarrow$ 10 numbers

$3k+2$ type: 2, 5, 8, ..., 29 $\rightarrow$ 10 numbers

$a^2 - b^2$ is divisible by 3 iff either both of a and b are divisible by 3 or none of a and b is divisible by 3.

So, number of favourable cases = ${}^{10}C_2 + {}^{20}C_2 = 235$

$$\therefore \text{Required probability} = \frac{235}{435} = \frac{47}{87}$$

19. Each ball can be put into any one of the three boxes. So, the total number of ways in which 12 balls can be put into three boxes is 3^{12} . Out of 12 balls, 3 balls can be chosen in ${}^{12}C_3$ ways for the first box. Now, remaining 9 balls can be put in the remaining 2 boxes in 2^9 ways. So, the total number of ways in which 3 balls are put in the first box and the remaining balls in other two boxes is ${}^{12}C_3 \times 2^9$. Hence, required probability is ${}^{12}C_3 \times 2^9 / 3^{12}$.

Exercise 9.3

1. $P(A) = 0.5, P(A \cap B) \leq 0.3$

So, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ gives

$$P(B) = P(A \cup B) - P(A) + P(A \cap B) \leq 1 + 0.3 - 0.5 = 0.8$$

$$[\because P(A \cup B) \leq 1]$$

Hence, $P(B) = 0.9$ is not possible

$$\begin{aligned}
2. \quad & P(A^c) = 1 - P(A) \text{ or } P(A) = 1 - P(A^c) = 1 - 2/3 = 1/3 \\
& P(A \cup B) = P(A) + P(B) - P(A \cap B) \\
& \Rightarrow 3/4 = 1/3 + P(B) - 1/4 \Rightarrow P(B) = 2/3 \\
& P(A \cap B^c) = P(A) - P(A \cap B) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \\
& P((A^c \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}
\end{aligned}$$

3. We have,

$$\begin{aligned}
& P(\bar{A} \cap \bar{B}) = \frac{1}{3} \\
\Rightarrow & P(\overline{A \cup B}) = \frac{1}{3} \\
\Rightarrow & 1 - P(A \cup B) = \frac{1}{3} \\
\text{or } & P(A \cup B) = \frac{2}{3} \\
\text{or } & P(A) + P(B) - P(A \cap B) = \frac{2}{3} \\
\Rightarrow & p + 2p - \frac{1}{2} = \frac{2}{3} \\
\text{or } & p = \frac{7}{18}
\end{aligned}$$

4. Consider the following events:

A : A student is passed in Mathematics

B : A student is passed in Statistics

Then,

$$P(A) = \frac{70}{125}, P(B) = \frac{55}{125}, P(A \cap B) = \frac{30}{125}$$

Required probability is

$$\begin{aligned}
& P(A \cap \bar{B}) + P(\bar{A} \cap B) \\
& = P(A) + P(B) - 2P(A \cap B) \\
& = \frac{70}{125} + \frac{55}{125} - \frac{60}{125} = \frac{65}{125} = \frac{13}{25}
\end{aligned}$$

5. Here,

$$P(R) = \frac{10}{100} = 0.1, P(F) = \frac{5}{100} = 0.05$$

$$P(F \cap R) = \frac{3}{100} = 0.03$$

Therefore, the required probability is

$$\begin{aligned}
P(R) + P(F) - 2P(F \cap R) &= 0.1 + 0.05 - 2(0.03) \\
&= 0.09
\end{aligned}$$

$$6. \quad P(C) = p; P(A) = 2p; P(B) = 2p$$

$$\therefore 5p = 1 \text{ or } p = 1/5$$

$$\text{So, } P(B \text{ or } C) = P(B) + P(C) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

7. Since $P(A \cup B \cup C) \geq 0.75$, we have

$$0.75 \leq P(A \cup B \cup C) \leq 1$$

$$\text{or } 0.75 \leq P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \leq 1$$

$$\text{or } 0.75 \leq 0.3 + 0.4 + 0.8 - 0.08 - P(B \cap C) - 0.28 + 0.09 \leq 1$$

$$\text{or } 0.75 \leq 1.23 - P(B \cap C) \leq 1$$

$$\text{or } -0.48 \leq -P(B \cap C) \leq -0.23$$

$$\text{or } 0.23 \leq P(B \cap C) \leq 0.48$$

Exercises

Singe Correct Answer Type

$$1. (1) \quad p^2 + 2p + 4p - 1 = 1 \quad (\text{Exhaustive})$$

$$p^2 + 6p - 2 = 0$$

$$\text{or } p = -3 \pm \sqrt{11} \\ = \sqrt{11} - 3$$

$$2. (2) \quad P(E) + P(E') = 1$$

$$\therefore 1 + \lambda + \lambda^2 + (1 + \lambda)^2 = 1$$

$$\Rightarrow 2\lambda^2 + 3\lambda + 1 = 0$$

$$\Rightarrow (2\lambda + 1)(\lambda + 1) = 0$$

$$\Rightarrow \lambda = -1, -\frac{1}{2}$$

For $\lambda = -1$, $P(E) = 1 + (-1) + (-1)^2 = 1$ (not possible)

$$\text{For } \lambda = -\frac{1}{2}, P(E) = 1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

3. (2) Since the sum of the noted numbers is 6, the number on the balls are either 1, 2, 3 or 2, 2, 2.

So, total number of cases = $3! + 1 = 7$

So, the required probability = $1/7$

4. (3) If A draws card higher than B , then the number of favorable cases is $(n-1) + (n-2) + \dots + 3 + 2 + 1$ (as when B draws card number 1, then A can draw any card from 2 to n and so on). Therefore, the required probability is

$$\frac{\frac{n(n-1)}{2}}{n^2} = \frac{n-1}{2n}$$

5. (1) L and W can be filled at 14 places in 2^{14} ways.

$$\therefore n(S) = 2^{14}$$

Now 13 L's and 1 W can be arranged at 14 places in 14 ways.

Hence, $n(A) = 14$.

$$\therefore p = \frac{14}{2^{14}} = \frac{7}{2^{13}}$$

6. (4) A person can have his/her birthday on any one of the seven days of the week. So 5 persons can have their birthdays in 7^5 ways. Out of 5, three persons can have their birthdays on days other than Sundays in 6^3 ways and other 2 on Sundays. Hence, the required probability is

$$= \frac{{}^5C_2 \times 6^3}{7^5} = \frac{10 \times 6^3}{7^5}$$

(Note that 2 persons can be selected out of 5 in 5C_2 ways.)

$$7. (1) \quad \text{Required probability} = \frac{\text{No. of favorable cases}}{\text{Total no. of exhaustive cases}}$$

$$= \frac{3}{3 \times 3 \times 3} = \frac{1}{9}$$

8. (4) The number of ways of arranging n numbers is $n!$ In each order obtained, we must now arrange the digits 1, 2, ..., k as group and the $n-k$ remaining digits. This can be done in $(n-k+1)!$ ways. Therefore, the probability for the required event is $(n-k+1)!/n!$

9. (2) The total number of cases is $11!/(2! \times 2!)$. The number of favorable cases is $[11!/(2! \times 2!)] - 9!$ Therefore, the required probability is

$$= 1 - \frac{9! \times 4}{11!} = \frac{53}{55}$$

10. (1) The number of ways in which 20 people can be divided into two equal groups is

$$n(S) = \frac{20!}{10! 10! 2!}$$

The number of ways in which 18 people can be divided into groups of 10 and 8 is

$$n(E) = \frac{18!}{10! 8!}$$

$$\therefore P(E) = \frac{18!}{10! 8!} \cdot \frac{10! 10! 2}{20!} = \frac{10 \times 9 \times 2}{20 \times 19} = \frac{9}{19}$$

11. (2) Total number of ways of distribution is 4^5 .

$$\therefore n(S) = 4^5$$

Total number of ways of distribution so that each child gets at least one game is

$$4^5 - {}^4C_1 3^5 + {}^4C_2 2^5 - {}^4C_3 = 1024 - 4 \times 243 + 6 \times 32 - 4 = 240$$

$$\therefore n(E) = 240$$

Therefore, the required probability is

$$\frac{n(E)}{n(S)} = \frac{240}{4^5} = \frac{15}{64}$$

12. (1) Out of 9 socks, 2 can be drawn in 9C_2 ways. Therefore, the total number of cases is 9C_2 . Two socks drawn from the drawer will match if either both are brown or both are blue. Therefore, favorable number of cases is ${}^5C_2 + {}^4C_2$. Hence, the required probability is

$$= \frac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \frac{4}{9}$$

13. (2) The total number of ways in which four-figure numbers can be formed is $4! = 24$. A number is divisible by 5 if at unit's place we have 5. Therefore, unit's place can be filled in one way and the remaining 3 places can be filled with the other digits in $3!$ ways. Hence, total number of numbers divisible by 5 is $3! = 6$. So, the required probability is $6/24 = 1/4$.

14. (1) Since each ball can be placed in any one of the 3 boxes, therefore there are 3 ways in which a ball can be placed in any one of the three boxes. Thus, there are 3^{12} ways in which 12 balls can be placed in 3 boxes. The number of ways in which 3 balls out of 12 can be put in the box is ${}^{12}C_3$. The remaining 9 balls can be placed in 2 boxes in 2^9 ways. So, required probability is

$$\frac{{}^{12}C_3}{3^{12}} 2^9 = \frac{110}{9} \left(\frac{2}{3}\right)^{10}$$

15. (4) The total number of ways of choosing 11 players out of 15 is ${}^{15}C_{11}$. A team of 11 players containing at least 3 bowlers can be chosen in the following mutually exclusive ways:

- (I) Three bowlers out of 5 bowlers and 8 other players out of the remaining 10 players.
- (II) Four bowlers out of 5 bowlers and 7 other players out of the remaining 10 players.
- (III) Five bowlers out of 5 bowlers and 6 other players out of the remaining 10 players.

So, required probability is

$$P(I) + P(II) + P(III)$$

$$= \frac{{}^5C_3 \times {}^{10}C_8}{{}^{15}C_{11}} + \frac{{}^5C_4 \times {}^{10}C_7}{{}^{15}C_{11}} + \frac{{}^5C_5 \times {}^{10}C_6}{{}^{15}C_{11}}$$

$$= \frac{1260}{1365} = \frac{12}{13}$$

16. (3) Required probability = $1 - P(G_5 \text{ and } G_7 \text{ are consecutive})$

$$= 1 - \frac{{}^4C_3}{{}^7C_5} = \frac{17}{21}$$

17. (2) Since lines are more than telegrams, ${}^N C_M$ are those lines where telegrams will be sent.

So, number of favourable cases = ${}^N C_M \times M!$

Total number of cases = N^M

[As first telegram can go in any one of n lines]

$$\text{So, required probability} = \frac{{}^N C_M \times M!}{N^M}$$

18. (2) There are 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The last two digits can be dialled in ${}^{10}P_2 = 90$ ways out of which only one way is favorable; thus, the required probability is $1/90$.

19. (1) For each toss, there are four choices:

- (i) A gets head, B gets head
- (ii) A gets tail, B gets head
- (iii) A gets head, B gets tail
- (iv) A gets tail, B gets tail

Thus, exhaustive number of ways is 4^{50} . Out of the four choices listed above, (iv) is not favorable to the required event in a toss. Therefore, favorable number of cases is 3^{50} . Hence, the required probability is $(3/4)^{50}$.

20. (2) Let A denote the event that there is an odd man out in a game. The total number of possible cases is 2^m . A person is odd man out if he is alone in getting a head or a tail.

The number of ways in which there is exactly one tail (head) and the rest are heads (tails) is ${}^m C_1 = m$. Thus, the number of favourable ways is $m + m = 2m$. Therefore,

$$P(A) = \frac{2m}{2^m} = \frac{m}{2^{m-1}}$$

21. (1) The total number of ways in which $2n$ boys can be divided into two equal groups is

$$= \frac{(2n)!}{(n!)^2 2!}$$

Now, the number of ways in which $2n - 2$ boys other than the two tallest boys can be divided into two equal groups is

$$= \frac{(2n-2)!}{((n-1)!)^2 2!}$$

Two tallest boys can be put in different groups in 2C_1 ways. Hence, the required probability is

$$2 \frac{(2n-2)!}{((n-1)!)^2 2!}$$

$$= \frac{(2n)!}{(n!)^2 2!}$$

$$= \frac{n}{2n-1}$$

22. (4) The total number of ways in which papers of 4 students can be checked by seven teachers is 7^4 . The number of ways of choosing two teachers out of 7 is 7C_2 . The number of ways in which they can check four papers is 2^4 . But this includes two ways in which all the papers will be checked by a single teacher. Therefore, the number of ways in which 4 papers can be checked by exactly two teachers is $2^4 - 2 = 14$. Therefore, the number of favorable ways is $({}^7C_2)(14) = (21)(14)$. Thus, the required probability is $(21)(14)/7^4 = 6/49$.

23. (2) We must have,

$$0 \leq P(A) \leq 1$$

$$\Rightarrow 0 \leq \frac{3x+1}{3} \leq 1$$

$$\Rightarrow \frac{-1}{3} \leq x \leq \frac{2}{3} \quad (1)$$

$$\text{and } 0 \leq P(B) \leq 1$$

$$\Rightarrow 0 \leq \frac{1-x}{4} \leq 1$$

$$\Rightarrow -3 \leq x \leq 1 \quad (2)$$

$$\text{and } 0 \leq P(A \cup B) = P(A) + P(B) \leq 1$$

$$\Rightarrow 0 \leq \frac{3x+1}{3} + \frac{1-x}{4} \leq 1$$

$$\Rightarrow \frac{-7}{9} \leq x \leq \frac{5}{9} \quad (3)$$

From (1), (2) and (3), we get $x \in \left[\frac{-1}{3}, \frac{5}{9} \right]$.

24. (3) We have,

$$x + \frac{100}{x} > 50$$

$$\text{or } x^2 + 100 > 50x$$

$$\text{or } (x-25)^2 > 525$$

$$\Rightarrow x-25 < -\sqrt{525} \text{ or } x-25 > \sqrt{525}$$

$$\Rightarrow x < 25 - \sqrt{525} \text{ or } x > 25 + \sqrt{525}$$

As x is a positive integer and $\sqrt{525} = 22.91$, we must have $x \leq 2$ or $x \geq 48$. Thus, the favorable number of cases is $2 + 53 = 55$. Hence, the required probability is $55/100 = 11/20$.

25. (3) Since it is known that each time a different digit appears, total number of cases is 6!

The sum in the first three throws is 12 if we get (1, 5, 6), (2, 4, 6) or (3, 4, 5) in any order.

So, number of cases for first three throws = $3 \times 3!$

The remaining three throws have $3!$ ways.

$\therefore$ Number of favourable cases = $3 \times 3! \times 3!$

$$\therefore \text{Required probability} = \frac{3 \times 3! \times 3!}{6!} = \frac{3}{20}$$

$$26. (3) x^2 + 2(a+4)x - 5a + 64 \geq 0$$

If $D < 0$, then

$$(a+4)^2 - (-5a+64) < 0$$

$$\text{or } a^2 + 13a - 48 < 0$$

$$\text{or } (a+16)(a-3) < 0$$

$$\Rightarrow -16 < a < 3 \Leftrightarrow -5 \leq a \leq 2$$

Then, the favorable cases is equal to the number of integers in the interval $[-5, 2]$, i.e., 8.

Total number of cases is equal to the number of integers in the interval $[-5, 30]$, i.e., 36.

Hence, the required probability is $8/36 = 2/9$.

27. (3) Given that 5 and 6 have appeared on two of the dice, the sample space reduces to $6^4 - 2 \times 5^4 + 4^4$ (inclusion-exclusion principle). Also, the number of favorable cases are $4! = 24$. So, the required probability is $24/302 = 12/151$.

28. (2) The prime digits are 2, 3, 5, 7. If we fix 2 at first place, then other $2n-1$ places are filled by all four digits. So the total number of cases is 4^{2n-1} .

Now, sum of 2 consecutive digits is prime when consecutive digits are (2, 3) or (2, 5). Then 2 will be fixed at all alternative places.

2		2		2		2	...	...	2	
---	--	---	--	---	--	---	-----	-----	---	--

So favorable number of cases is 2^n . Therefore, probability is

$$\frac{2^n}{4^{2n-1}} = 2^n 2^{-4n+2} = 2^2 2^{-3n} = \frac{4}{2^{3n}}$$

29. (3) When 4 points are selected, we get one intersecting point.

So, probability is

$$\frac{{}^nC_4}{{}^nC_{2-n} {}^nC_2}$$

30. (4) Three-digit numbers are 100, 101, ..., 999. Total number of such numbers is 900. The three-digit numbers (which have all same digits) are 111, 222, 333, ..., 999. Favorable number of cases is 9. Therefore, the required probability is $9/900 = 1/100$.

31. (3) Given,

$$7a - 9b = 0 \text{ or } b = \frac{7}{9}a$$

Hence, number of pairs (a, b) can be (9, 7); (18, 14); (27, 21); (36, 28). Hence, the required probability is $4/39 C_2 = 4/741$.

32. (3) The total number of mapping is n^n . The number of one-one mapping is $n!$. Hence, the probability is

$$\frac{n!}{n^n} = \frac{3}{32} = \frac{4!}{4^4}$$

Comparing, we get $n = 4$.

33. (1) The number of composite numbers in 1 to 30 is $n(S) = 19$.

The number of composite numbers when divided by 5 leaves a remainder is $n(E) = 14$. Therefore, the required probability is $14/19$.

34. (2) Team totals must be 0, 1, 2, ..., 39. Let the teams be $T_1 T_2, \dots, T_{40}$, so that T_i loses to T_j for $i < j$. In other words, this order uniquely determines the result of every game. There are $40!$ such orders and 780 games, so 2^{780} possible outcomes for the games. Hence, the probability is $40!/2^{780}$.

35. (1) We have,

$$n(S) = {}^{64}C_3$$

Let 'E' be the event of selecting 3 squares which form the letter 'L'.

The number of ways of selecting squares consisting of 4 unit squares is $7 \times 7 = 49$.

Each square with 4 unit squares form 4 L-shapes consisting of 3 unit squares. Therefore,

$$n(E) = 7 \times 7 \times 4 = 196$$

$$\therefore P(E) = \frac{196}{{}^{64}C_3}$$

36. (1) The total number of ways of making the second draw is ${}^{10}C_5$.

The number of draws of 5 balls containing 2 balls common with first draw of 6 balls is ${}^6C_2 {}^4C_3$. Therefore, the probability is

$$\frac{{}^6C_2 {}^4C_3}{{}^{10}C_5} = \frac{5}{21}$$

37. (2) A number has exactly 3 factors if the number is squares of a prime number. Squares of 11, 13, 17, 19, 23, 29, 31 are 3-digit numbers. Hence, the required probability is $7/900$.

38. (3) Suppose, there exist three rational points or more on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Therefore, if (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) are those three points, then

$$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0 \quad (1)$$

$$x_2^2 + y_2^2 + 2gx_2 + 2fy_2 + c = 0 \quad (2)$$

$$x_3^2 + y_3^2 + 2gx_3 + 2fy_3 + c = 0 \quad (3)$$

Solving Eqs. (1), (2) and (3), we will get g, f, c as rational. Thus, center of the circle $(-g, -f)$ is a rational point. Therefore, both the coordinates of the center are rational numbers. Obviously, the possible values of p are 1, 2. Similarly, the possible values of q are 1, 2. Thus, for this case, (p, q) may be chosen in 2×2 , i.e., 4 ways. Now, (p, q) can be, without restriction, chosen in 6×6 , i.e., 36 ways.

Hence, the probability that at the most two rational points exist on the circle is $(36 - 4)/36 = 32/36 = 8/9$.

39. (1) The total number of ways of selecting 3 integers from 20 natural numbers is ${}^{20}C_3 = 1140$. Their product is a multiple of 3 means at least one number is divisible by 3. The numbers which are divisible by 3 are 3, 6, 9, 12, 15, 18 and the number of ways of selecting at least one of them is ${}^6C_1 \times {}^{14}C_2 + {}^6C_2 \times {}^{14}C_1 + {}^6C_3 = 776$. Hence, the required probability is $776/1140 = 194/285$.

40. (3) We have,

$$n(S) = 5^5$$

For computing favorable outcomes, 2 boxes which are to remain empty, can be selected in 5C_2 ways and 5 marbles can be placed in the remaining 3 boxes in groups of 221 or 311 in

$$3! \left[\frac{5!}{2!2!2!} + \frac{5!}{3!2!} \right] = 150 \text{ ways}$$

$$\Rightarrow n(A) = {}^5C_2 \times 150$$

Hence,

$$P(E) = {}^5C_2 \times \frac{150}{5^5} = \frac{60}{125} = \frac{12}{25}$$

41. (1) $n(S) = {}^{10}C_7 = 120$

$$n(E) = {}^5C_4 \times {}^3C_2 \times {}^2C_1$$

$$\therefore P(E) = \frac{5 \times 3 \times 2}{120} = \frac{1}{4}$$

42. (4) Since there are r cars in N places, total number of selection of places out of $N - 1$ places for $r - 1$ cars (except the owner's car) is

$${}^{N-1}C_{r-1} = \frac{(N-1)!}{(r-1)!(N-r)!}$$

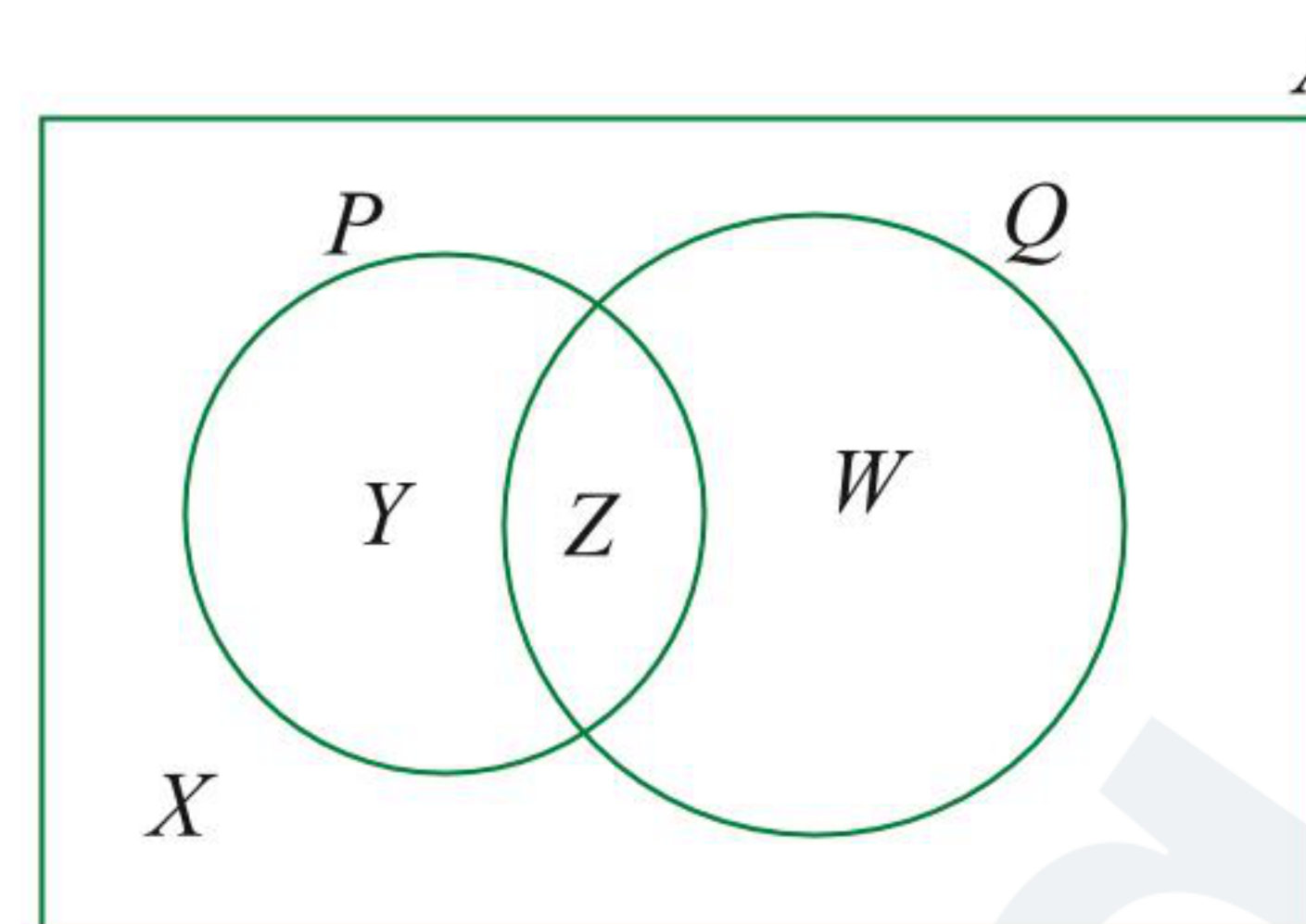
If neighboring places are empty, then $r - 1$ cars must be parked in $N - 3$ places. So, the favorable number of cases is

$${}^{N-3}C_{r-1} = \frac{(N-3)!}{(r-1)!(N-r-2)!}$$

Therefore, the required probability is

$$\begin{aligned} &= \frac{(N-3)!}{(r-1)!(N-r-2)!} \times \frac{(r-1)!(N-r)!}{(N-1)!} \\ &= \frac{(N-r)(N-r-1)}{(N-1)(N-2)} = \frac{{}^{N-r}C_2}{{}^{N-1}C_2} \end{aligned}$$

43. (3)



Sets P and Q are subsets of set A . Sets P and Q are dividing set A in four regions X, Y, Z and W .

$\therefore$ Total number of cases

= Number of ways in which n elements can be distributed in four regions

$$= 4^n$$

If set $(P \cap Q)$ has exactly m elements, we can select these m elements in nC_m ways. Remaining $n - m$ elements can be distributed in the regions X, Y and W in 3^{n-m} ways.

$\therefore$ Number of favourable ways = ${}^nC_m \times 3^{n-m}$

$$\therefore \text{Required probability} = \frac{{}^nC_m \times 3^{n-m}}{4^n}$$

44. (3) $f'(x) = 3x^2 + 2ax + b$

$y = f(x)$ is increasing

$\Rightarrow f'(x) \geq 0, \forall x$ and for $f'(x) = 0$ should not form an interval

$$\Rightarrow (2a)^2 - 4 \times 3 \times b \leq 0 \Rightarrow a^2 - 3b \leq 0$$

This is true for exactly 16 ordered pairs (a, b) , $1 \leq a, b \leq 6$, namely $(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)$, and $(4, 6)$. Thus, the required probability is $16/36 = 4/9$

45. (3) Given limit,

$$\lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{2}{x}}$$

$$\begin{aligned} &= \left(\lim_{x \rightarrow 0} \left(1 + \frac{a^x + b^x - 2}{2} \right)^{\frac{2}{a^x + b^x - 2}} \right)^{\lim_{x \rightarrow 0} \left(\frac{a^x - 1 + b^x - 1}{x} \right)} \\ &= e^{\log ab} = ab = 6 \end{aligned}$$

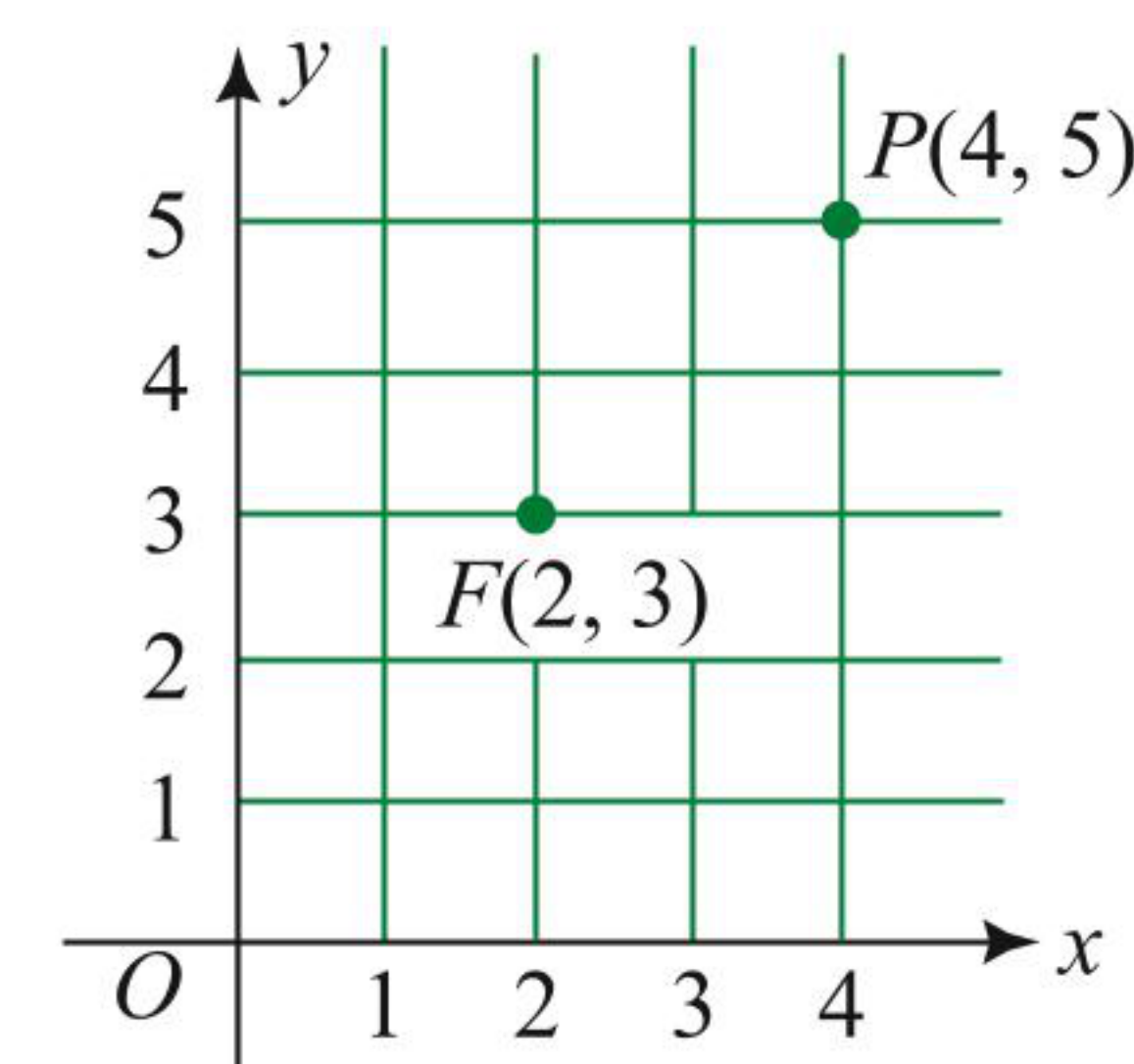
Total number of possible ways in which a, b can take values is $6 \times 6 = 36$. Total possible ways are $(1, 6), (6, 1), (2, 3), (3, 2)$. The total number of possible ways is 4. Hence, the required probability is $4/36 = 1/9$.

46. (2) $n(S) = \frac{9!}{4! \cdot 5!} = 126$

$n(A) = O \text{ to } F \text{ and } F \text{ to } P$

$$= \frac{5!}{2! \times 3!} \times \frac{4!}{2! \times 2!} = 60$$

$$\Rightarrow P(A) = \frac{60}{126} = \frac{10}{21}$$



Multiple Correct Answers Type

1. (1), (2), (3), (4)

We have,

$P(\text{exactly one of } A, B \text{ occurs})$

$$= P[(A \cap \bar{B}) \cup (\bar{A} \cap B)]$$

$$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$\begin{aligned}
&= P(A) - P(A \cap B) + P(B) - P(A \cap B) \\
&= P(A) + P(B) - 2P(A \cap B) \\
&= P(A \cup B) - P(A \cap B)
\end{aligned}$$

Also,

$$\begin{aligned}
&P(\text{exactly one of } A, B \text{ occurs}) \\
&= [1 - P(\bar{A} \cap \bar{B})] - [1 - P(\bar{A} \cup \bar{B})] \\
&= P(\bar{A} \cup \bar{B}) - P(\bar{A} \cap \bar{B}) = P(\bar{A}) + P(\bar{B}) - 2P(\bar{A} \cap \bar{B})
\end{aligned}$$

2. (2), (3), (4)

Roots of $x^2 + px + q = 0$ will be real if $p^2 \geq 4q$.

The possible selections are as follows:

p	q
1	—
2	1
3	1, 2
4	1, 2, 3, 4
5	1, 2, 3, 4, 5, 6
6	1, 2, ..., 9
7	1, 2, ..., 10
8	1, 2, ..., 10
9	1, 2, ..., 10
10	1, 2, ..., 10
Total	62

Therefore, number of favourable ways is 62 and total number of ways is $10^2 = 100$. Hence, the required probability is $62/100 = 31/50$.

The probability that the roots are imaginary is $1 - 31/50 = 19/50$.

Roots are equal when $(p, q) \equiv (2, 1), (4, 4), (6, 9)$. The probability that the roots are real and equal is $3/50$. Hence, probability that the roots are real and distinct is $3/5$.

3. (1), (2), (3), (4)

$$\begin{aligned}
&A \subseteq A \cup B \\
\Rightarrow &P(A) \leq P(A \cup B) \\
\Rightarrow &P(A \cup B) \geq \frac{3}{4}
\end{aligned}$$

$$\begin{aligned}
\text{Also, } P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\
&\geq P(A) + P(B) - 1 \\
&= \frac{3}{4} + \frac{5}{8} - 1 = \frac{3}{8}
\end{aligned}$$

Now, $A \cap B \subseteq B$

$$\Rightarrow P(A \cap B) \leq P(B) = \frac{5}{8}$$

$$\therefore \frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8} \quad (1)$$

$$\text{and } P(A \cap B') = P(A) - P(A \cap B)$$

$$\text{or } \frac{3}{4} - \frac{5}{8} \leq P(A \cap B') \leq \frac{3}{4} - \frac{3}{8}$$

$$\text{or } \frac{1}{8} \leq P(A \cap B') \leq \frac{3}{8}$$

$$\therefore P(A \cap B) = P(B) - P(A' \cap B) \quad [\text{Using Eq. (1)}]$$

$$\text{or } \frac{3}{8} \leq P(B) - P(A' \cap B) \leq \frac{5}{8}$$

$$\text{or } 0 \leq P(A' \cap B) \leq \frac{1}{4}$$

4. (1), (3)

Given that A and B are mutually exclusive events.

$$\begin{aligned}
\therefore &A \cap B = \phi \\
\Rightarrow &A \subseteq \bar{B} \text{ and } B \subseteq \bar{A} \\
\Rightarrow &P(A) \leq P(\bar{B}) \text{ and } P(B) \leq P(\bar{A})
\end{aligned}$$

5. (1), (3), (4)

$$\begin{aligned}
P(E) &= \frac{{}^{2n}C_n}{2^{2n}} = \frac{(2n)!}{n! n! 2^n 2^n} \\
&= \frac{1 \times 2 \times 3 \times \cdots \times (2n)}{n! n! 2^n 2^n} \\
&= \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{n! 2^n}
\end{aligned}$$

Now,

$$\begin{aligned}
\prod_{r=1}^n \left(\frac{2r-1}{2r} \right) &= \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2 \times 4 \times 6 \times \cdots \times (2n)} \\
&= \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{(1 \times 2 \times 3 \times \cdots \times n) 2^n}
\end{aligned}$$

$$\begin{aligned}
\sum_{r=0}^n \left(\frac{{}^nC_r}{2^n} \right)^2 &= \frac{1}{2^n 2^n} \sum_{r=0}^n ({}^nC_r)^2 \\
&= \frac{1}{2^n 2^n} {}^{2n}C_n
\end{aligned}$$

$$\text{Also, } \frac{\sum_{r=0}^n ({}^nC_r)^2}{\left(\sum_{r=0}^{2n} {}^{2n}C_r \right)} = \frac{{}^{2n}C_n}{2^{2n}}$$

6. (1), (3)

Let p_1, p_2 be the chances of happening of the first and second events, respectively. Then according to the given conditions, we have

$$\begin{aligned}
p_1 &= p_2^2 \\
\text{and } \frac{1-p_1}{p_1} &= \left(\frac{1-p_2}{p_2} \right)^3
\end{aligned}$$

Hence,

$$\frac{1-p_2^2}{p_2^2} = \left(\frac{1-p_2}{p_2} \right)^3 \quad \text{or } p_2(1+p_2) = (1-p_2)^2$$

$$\text{or } 3p_2 = 1 \text{ or } p_2 = \frac{1}{3} \text{ and so } p_1 = \frac{1}{9}$$

7. (1), (2)

Let the number of red and blue balls be r and b , respectively.

Then, the probability of drawing two red balls is

$$p_1 = \frac{{}^rC_2}{{}^{r+b}C_2} = \frac{r(r-1)}{(r+b)(r+b-1)}$$

The probability of drawing two blue balls is

$$p_2 = \frac{{}^bC_2}{{}^{r+b}C_2} = \frac{b(b-1)}{(r+b)(r+b-1)}$$

The probability of drawing one red and one blue ball is

$$p_3 = \frac{{}^rC_1 \times {}^bC_1}{{}^{r+b}C_2} = \frac{2br}{(r+b)(r+b-1)}$$

By hypothesis, $p_1 = 5p_2$ and $p_3 = 6p_2$. Therefore,
 $r(r-1) = 5b(b-1)$ and $2br = 6b(b-1)$
 $\Rightarrow r = 6, b = 3$

8. (1), (2), (3)

Here total number of cases is ${}^8C_2 = 28$.

1. Number of favorable case is 13.

For $2 \rightarrow 6$ choices

For $1 \rightarrow 7$ choices

2. For $6 \rightarrow 5$ choices

For $7 \rightarrow 6$ choices

For $8 \rightarrow 7$ choices

3. For $1 \rightarrow 4$ choices (2, 4, 6, 8)

For $2 \rightarrow 6$ choices (3, 4, 5, 6, 7, 8)

For $3 \rightarrow 3$ choices (4, 6, 8)

For $4 \rightarrow 4$ choices (5, 6, 7, 8)

For $5 \rightarrow 2$ choices (6, 8)

For $6 \rightarrow 2$ choices (7, 8)

For $7 \rightarrow 1$ choice (8)

Alternate solution:

$$\text{a. } \frac{{}^8C_2 - {}^6C_2}{{}^8C_2} = \frac{13}{28}$$

$$\text{b. } \frac{{}^8C_2 - {}^5C_2}{{}^8C_2} = \frac{9}{14}$$

$$\text{c. } \frac{{}^8C_2 - {}^4C_2}{{}^8C_2} = \frac{11}{14}$$

Linked Comprehension Type

1. (3), 2. (4), 3. (4)

Let in 8 coupons S, U, R, F appears x_1, x_2, x_3, x_4 times. Then $x_1 + x_2 + x_3 + x_4 = 8$, where $x_1, x_2, x_3, x_4 \geq 0$.

We have to find non-negative integral solutions of the equation. The total number of such solutions is

$${}^{8+4-1}C_{4-1} = {}^{11}C_3 = 165.$$

If a person gets at least one free packet, then he must get each coupon at least once, which is equal to number of positive integral solutions of the equation. The number of such solutions is ${}^{8-1}C_{4-1} = {}^7C_3 = 35$. Then, the probability that he gets exactly one free packet is

$$(35 - 1)/165 = 34/165$$

The probability that he gets two free packets is $1/{}^{11}C_3 = 1/165$.

4. (3) x can be 2, 3, 4, 5, 6. The number of ways in which sum of 2, 3, 4, 5, 6 can occur is given by the coefficients of x^2, x^3, x^4, x^5, x^6 in

$$(3x + 2x^2 + x^3)(x + 2x^2 + 3x^3) = 3x^2 + 8x^3 + 14x^4 + 8x^5 + 3x^6$$

This shows that sum that occurs most often is 4.

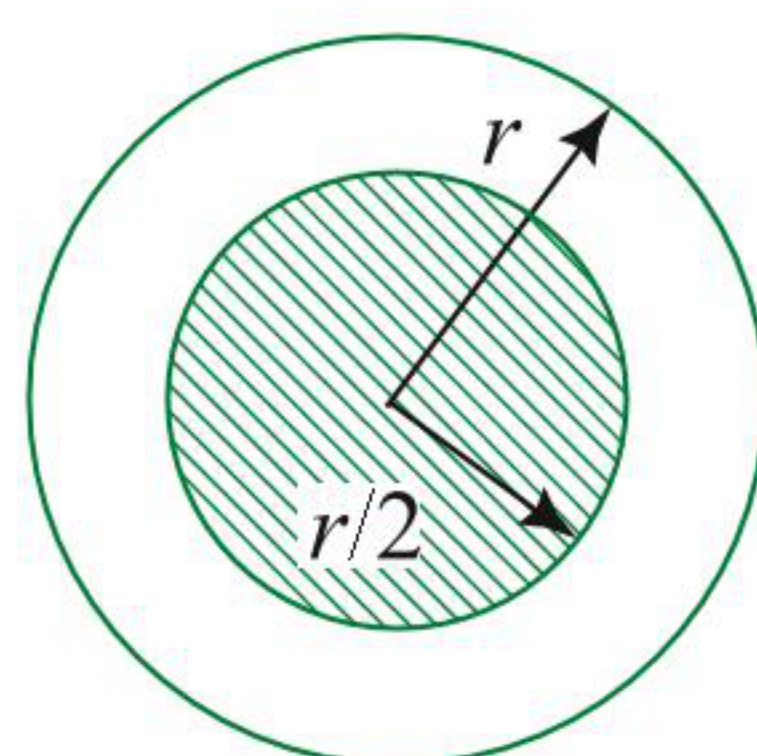
5. (4) Sum that occurs for minimum times is 2 or 6.

6. (3) The number of ways in which different sums can occur is $(3 + 2 + 1)(1 + 2 + 3) = 36$. The probability of 4 is $14/36 = 7/18$.

7. (2) 8. (2), 9. (4)

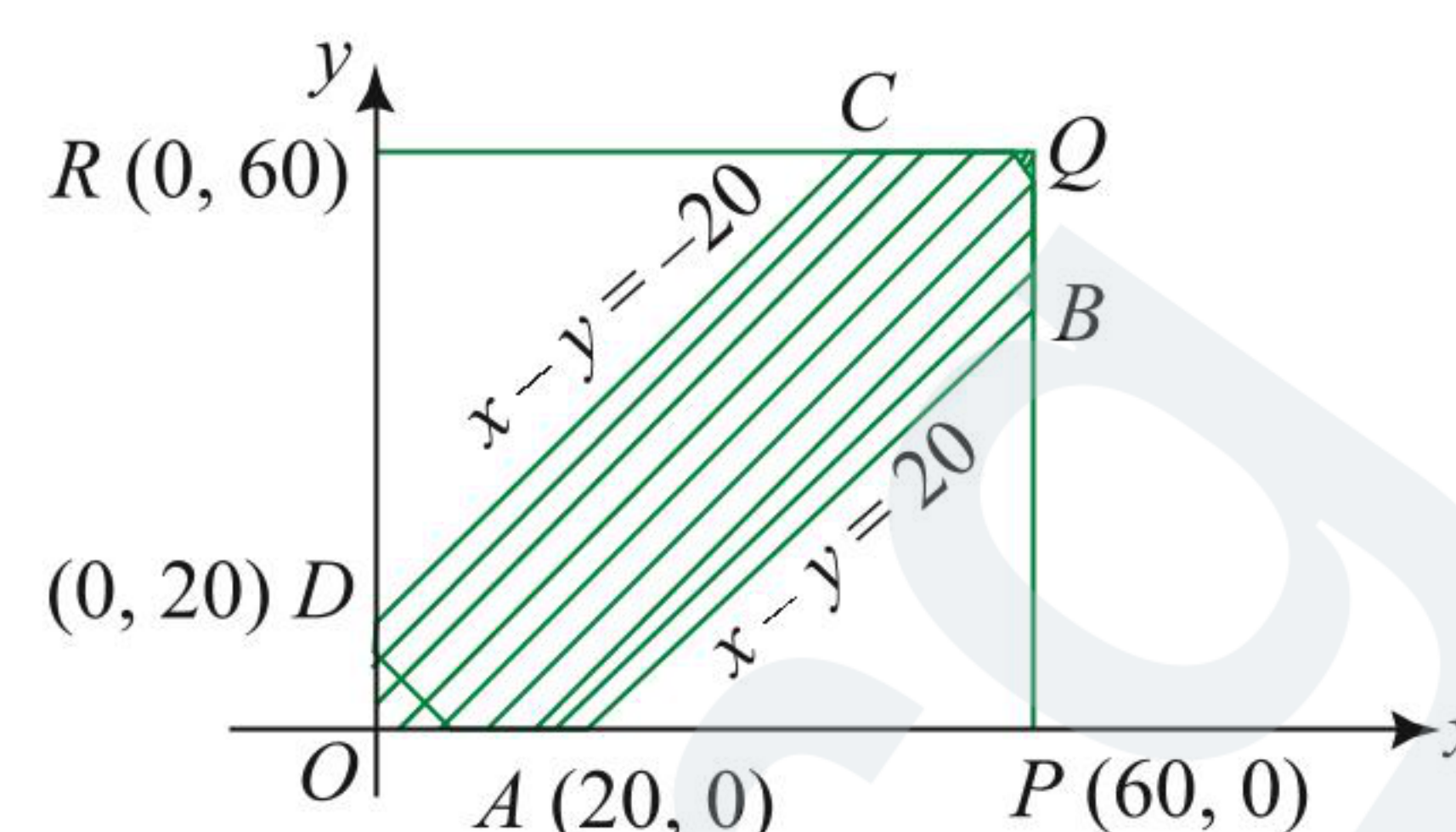
Clearly, cube at the centre is the only cube which is not painted. On each edge, we have one cube painted on two sides. So, there are twelve cubes painted on two sides.

10. (1) For the favorable cases, the points should lie inside the concentric circle of radius $r/2$. So the desired probability is given by



$$\frac{\text{Area of smaller circle}}{\text{Area of larger circle}} = \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$$

11. (4)



Let A and B arrive at the place of their meeting ' a ' minutes and ' b ' minutes after 5 pm. Their meeting is possible only if

$$|a - b| \leq 20 \quad (1)$$

Clearly, $0 \leq a \leq 60$ and $0 \leq b \leq 60$. Therefore, a and b can be selected as an ordered pair (a, b) from the set $[0, 60] \times [0, 60]$.

Alternatively, it is equivalent to select a point (a, b) from the square $OPQR$, where P is $(60, 0)$ and R is $(0, 60)$ in the Cartesian plane. Now,

$$|a - b| \leq 20 \Rightarrow -20 \leq a - b \leq 20$$

Therefore, points (a, b) satisfy the equation $-20 \leq x - y \leq 20$.

Hence, favorable condition is equivalent to selecting a point from the region bounded by $y \leq x + 20$ and $y \geq x - 20$. Therefore, the required probability is

$$\begin{aligned} \frac{\text{Area of } OABQCDO}{\text{Area of square } OPQR} &= \frac{[\text{Ar}(OPQR) - 2\text{Ar}(\Delta APB)]}{\text{Ar}(OPQR)} \\ &= \frac{60 \times 60 - \frac{2}{2} \times 40 \times 40}{60 \times 60} = \frac{5}{9} \end{aligned}$$

12. (1) The total number of ways of painting first column when colors are not alternating is $2^8 - 2$. The total number of ways when no column has alternating colors is $(2^8 - 2)^8/2^{64}$.

13. (1) The number of ways the square has equal number of red and black squares is ${}^{64}C_{32}$. Hence, the required probability is ${}^{64}C_{32}/2^{64}$.

14. (2) This is possible only when the column are alternating red and black. Hence, the required probability is $2/2^{64} = 1/2^{63}$.

Matrix Match Type

1. **a** $\rightarrow$ **r**; **b** $\rightarrow$ **p**; **c** $\rightarrow$ **q**; **d** $\rightarrow$ **s**.

a. The required event will occur if last digit in all the chosen numbers is 1, 3, 7, or 9. Therefore, the required probability is $(4/10)^n$.

b. The required probability is equal to the probability that the last digit is 2, 4, 6, 8 and is given by

$$P(\text{last digit is } 1, 2, 3, 4, 6, 7, 8, 9)$$

$$-P(\text{last digit is } 1, 3, 7, 9) = \frac{8^n - 4^n}{10^n}$$

$$\text{c. } P(1, 3, 5, 7, 9) - P(1, 3, 7, 9) = \frac{5^n - 4^n}{10^n}$$

d. The required probability is

$$\begin{aligned} P(0, 5) - P(5) &= \frac{(10^n - 8^n) - (5^n - 4^n)}{10^n} \\ &= \frac{10^n - 8^n - 5^n + 4^n}{10^n} \end{aligned}$$

2. (3) $n(s) = {}^{100}C_3$
- a. $2b = a + c = \text{even}$
This means that a and c are both even or both odd.
 $n(E) = {}^{50}C_2 + {}^{50}C_2 = 50 \times 49$
- b. Taking $r = 2, 3, \dots, 10$,
 a, b, c can be in GP in 53 ways.
- c. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in GP $\Rightarrow a, b, c$ are in GP
- d. $P(a + b + c \text{ is even}) = \left(\frac{{}^{50}C_3 + {}^{50}C_1 \times {}^{50}C_2}{{}^{100}C_3} \right) = \frac{1}{2}$

Numerical Value Type

1. (2) Total number of cases $= n(S) = 6!$
Now sum the given digits is $1 + 2 + 3 + 4 + 5 + 6 = 21$, which is divisible by 3.
Now we have to form the number which is divisible by 6, then we have to ensure that the digit in unit place is even.
 $\Rightarrow$ Favorable cases $= n(A) = 3 \cdot 5!$
Hence, $P(A) = \frac{3 \cdot 5!}{6!} = \frac{1}{2}$.
2. (6) Total number of cases $n(S) = 6^3 = 216$
Product is prime only when two outcomes are 1 and the third is prime, i.e., 2, 3, 5.
If it is 2, 1, 1, then the number of cases is 3.
Similarly, 3 cases for 3, 1, 1 and 5, 1, 1 each
Hence, favorable cases $= 9$.
Hence, required probability $p = \frac{1}{24}$
 $\Rightarrow \frac{1}{4p} = 6$
3. (4) Let the number of green socks be $x > 0$.
 E : Two socks drawn are of the same color
 $\Rightarrow P(E) = P(RR \text{ or } BB \text{ or } WW \text{ or } GG)$
$$= \frac{3}{{}^{6+x}C_2} + \frac{{}^xC_2}{{}^{6+x}C_2}$$

$$= \frac{6}{(x+6)(x+5)} + \frac{x(x-1)}{(x+6)(x+5)} = \frac{1}{5} \text{ (given)}$$

 $\Rightarrow 5(x^2 - x + 6) = x^2 + 11x + 30$
or $4x^2 - 16x = 0$
or $x = 4$
4. (5) $P(n) = Kn^2$
Given $P(1) = K, P(2) = 2^2K, P(3) = 3^2K, P(4) = 4^2K,$
 $P(5) = 5^2K, P(6) = 6^2K$
So, total probability $= 91K$
 $\Rightarrow 1 = 91K$
 $\therefore K = \frac{1}{91}$
Therefore, $P(1) = \frac{1}{91}; P(2) = \frac{4}{91}$ and so on.

Let three events A, B and C be defined as

$$A: a < b$$

$$B: a = b$$

$$C: a > b$$

By symmetry, $P(A) = P(C)$

$$\text{Also, } P(A) + P(B) + P(C) = 1$$

$$\begin{aligned} \text{Since } P(B) &= \sum_{i=1}^6 [P(i)]^2 \\ &= \left[\frac{1+16+81+256+625+1296}{91 \times 91} \right] \\ &= \frac{2275}{91 \times 91} = \frac{25}{91} \end{aligned}$$

$$\text{Now, } 2P(A) + P(B) = 1$$

$$\Rightarrow P(A) = \frac{1}{2} [1 - P(B)] = \frac{33}{91}$$

5. (6) Given S_2 reaches the semi-finals.

Since all other players $(2^n - 1)$ are equally likely to win the finals with probability p .

$$\therefore (2^n - 1)p + \frac{1}{4} = 1$$

$$\Rightarrow (2^n - 1)p = \frac{3}{4}$$

$$\Rightarrow p = \frac{3}{4(2^n - 1)}$$

$$\text{If } p = \frac{1}{84}, \text{ then}$$

$$\therefore \frac{1}{84} = \frac{3}{4(2^n - 1)}$$

$$\Rightarrow 2^n - 1 = 63$$

$$\Rightarrow 2^n = 64$$

$$\Rightarrow n = 6$$

6. (4) Total ways of distribution $= n(S) = 4^5$

Total ways of distribution so that each child get at least one game

$$\begin{aligned} n(E) &= 4^5 - {}^4C_1 3^5 + {}^4C_2 2^5 - {}^4C_3 \\ &= 1024 - 4 \times 243 + 6 \times 32 - 4 \\ &= 240 \end{aligned}$$

$$\text{Required probability } p = \frac{n(E)}{n(S)} = \frac{240}{4^5} = \frac{15}{64}$$

7. (7) Number of ways of drawing 7 balls (second draw) $= {}^{10}C_7$

For each set of 7 balls of the second draw, 3 must be common to the set of 5 balls of the first draw, i.e., 2 other balls can be drawn in 3C_2 ways.

Thus, for each set of 7 balls of the second draw, there are ${}^7C_3 \times {}^3C_2$ ways of making the first draw so that there are 3 balls common.

$$\begin{aligned} \therefore \text{Probability of getting 3 balls in common} &= \frac{{}^7C_3 \times {}^3C_2}{{}^{10}C_7} \\ &= \frac{7}{8} \end{aligned}$$

Archives

JEE ADVANCED

Singe Correct Answer Type

1. (3) $r_1, r_2, r_3 \in \{1, 2, 3, 4, 5, 6\}$

r_1, r_2, r_3 are of the form $3k, 3k+1, 3k+2$

$$\text{Required probability} = \frac{3! \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{6 \times 6 \times 6} = \frac{6 \times 8}{216} = \frac{2}{9}$$

2. (1) 3 Boys and 2 Girls ...

(1) B (2) B (3) B (4)

Girl can't occupy 4th position.

Either girls can occupy 2 of 1, 2, 3 position or they can both be at position (1) or (2).

Hence, total number of ways in which girls can be seated

$$= {}^3C_2 \times 2! \times 3! + {}^2C_1 \times 2! \times 3! = 36 + 24 = 60.$$

Number of ways in which 3 boys and 2 girls can be seated = 5!

$$\therefore \text{Required probability} = \frac{60}{5!} = \frac{1}{2}$$

3. (3) $x + y + z = 10$

$$\begin{aligned} \text{Total number of non-negative solutions} &= {}^{10+3-1}C_{3-1} \\ &= {}^{12}C_2 = 66 \end{aligned}$$

Now let $z = 2n$

$$\therefore x + y = 10 - 2n; n \geq 0$$

Number of non-negative solutions of above equation is

$$11 + 9 + 7 + 5 + 3 + 1 = 36.$$

$$\therefore \text{Required probability} = \frac{36}{66} = \frac{6}{11}$$

Multiple Correct Answer Type**1. (1), (3)**

	Bag B_1	Bag B_2	Bag B_3
Red Balls	5	3	5
Green Balls	5	5	3
Total	10	8	8

 $P(\text{Ball is Green})$

$$= P(B_1)P(G/B_1) + P(B_2)P(G/B_2) + P(B_3)P(G/B_3)$$

$$= \frac{3}{10} \times \frac{5}{10} + \frac{3}{10} \times \frac{5}{8} + \frac{4}{10} \times \frac{3}{8} = \frac{39}{80}$$

$$P(\text{Ball chosen is Green/Ball is from 3rd Bag}) = \frac{3}{8}$$

 $P(\text{Ball is from 3rd Bag/Ball chosen is Green})$

$$= \frac{P(B_3)P(G/B_3)}{P(\text{Ball is green})} = \frac{\frac{4}{10} \times \frac{3}{8}}{\frac{39}{80}} = \frac{4}{13}$$

2. (1), (2), (3)

$$P(E) = \frac{1}{8}; P(F) = \frac{1}{6}; P(G) = \frac{1}{4}; P(E \cap F \cap G) = \frac{1}{10}$$

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G)$$

$$- P(G \cap E) + P(E \cap F \cap G)$$

$$= \frac{1}{8} + \frac{1}{6} + \frac{1}{4} - \sum P(E \cap F) + \frac{1}{10}$$

$$= \frac{13}{24} + \frac{1}{10} - \sum P(E \cap F)$$

$$\therefore P(E \cup F \cup G) \leq \frac{13}{24}$$

So, option (3) is true.

$$\text{Now, } P(E^C \cap F^C \cap G^C) = 1 - P(E \cup F \cup G) \geq 1 - \frac{13}{24}$$

$$\therefore P(E^C \cap F^C \cap G^C) \geq \frac{11}{24}$$

So, option (4) is not true.

$$\text{Now, } P(E) \geq P(E \cap F \cap G^C) + P(E \cap F \cap G)$$

$$\Rightarrow \frac{1}{8} \geq P(E \cap F \cap G^C) + \frac{1}{10}$$

$$\Rightarrow P(E \cap F \cap G^C) \leq \frac{1}{40}$$

So, option (1) is true.

$$P(F) \geq P(E^C \cap F \cap G) + P(E \cap F \cap G)$$

$$\Rightarrow \frac{1}{15} \geq P(E^C \cap F \cap G)$$

So, option (2) is true.

Linked Comprehension Type**1. (2)** For, $x_1 + x_2 + x_3 = \text{odd}$.**Case I:** One odd, two even

(OEE) or (EOE) or (EEO)

$$\text{Total number of ways} = 2 \times 2 \times 3 + 1 \times 3 \times 3 + 1 \times 2 \times 4 = 29.$$

Case II: All three odd

$$\text{Number of ways} = 2 \times 3 \times 4 = 24$$

$$\therefore \text{Favorable ways} = 53$$

$$\therefore \text{Required probability} = \frac{53}{3 \times 5 \times 7} = \frac{53}{105}$$

2. (3) Here, $2x_2 = x_1 + x_3$

$$\Rightarrow x_1 + x_3 = \text{even}$$

So, either x_1 and x_3 are both odd or both even.

$$\text{Hence, number of favorable ways} = {}^2C_1 \cdot {}^4C_1 + {}^1C_1 \cdot {}^3C_1 = 11.$$

$$\text{Therefore, required probability is } \frac{11}{105}.$$

3. (1) Total number of cases = 5!Since S_1 gets seat R_1 and none of the other gets previously allotted seat, we have derangement of 4 students.

So, required probability

$$= \frac{4! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right)}{5!} = \frac{9}{120} = \frac{3}{40}$$

4. (3) Event $T_1 \cap T_2 \cap T_3 \cap T_4$: No two students with consecutive index sit adjacent.

So, we have following possible arrangements:

13524, 14253

24135, 24153, 25314

31425, 31524, 35142, 35241

41352, 42531, 42513

52413, 53142

Thus, favorable cases = 14

$$\text{Hence, required probability} = \frac{14}{120} = \frac{7}{60}$$

Numerical Value Type**Solution to Questions 1 and 2:****1. (76.25), 2. (24.5)** p_1 = probability that the maximum of chosen number is at least 81

= 1 - probability that all the drawn numbers are less than or equal to 80.

$$= 1 - \frac{80}{100} \times \frac{80}{100} \times \frac{80}{100}$$

$$= 1 - \frac{64}{125} = \frac{61}{125}$$

$$\therefore \frac{625}{4} p_1 = \frac{625}{4} \times \frac{61}{125} = 76.25$$

p_2 = probability that the minimum of chosen number is at most 40

= 1 – probability that all the drawn numbers are more than or equal to 41.

$$= 1 - \frac{60}{100} \times \frac{60}{100} \times \frac{60}{100}$$

$$= 1 - \frac{27}{125} = \frac{98}{125}$$

$$\therefore \frac{125}{4} p_2 = \frac{125}{4} \times \frac{98}{125} = 24.5$$

3. (214) $n(3k) = \left[\frac{2000}{3} \right] = 666$, where $[.]$ represents greatest integer function.

$$n(7k) = \left[\frac{2000}{7} \right] = 285$$

$$n(21k) = \left[\frac{2000}{21} \right] = 95$$

$$\therefore p = p(3k \cup 7k) = p(3k) + p(7k) - p(3k \cap 7k)$$

$$= \frac{666}{2000} + \frac{285}{2000} - \frac{95}{2000} = \frac{856}{2000}$$

$$\therefore 500 p = \frac{856}{4} = 214$$